

INFILTRATION PREDICTION BASED
ON IN-SITU MEASUREMENTS OF
SOIL-WATER PROPERTIES

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ABSTRACT

In watershed simulation we need a physically based infiltration equation which can accommodate variation in antecedent soil water content and also spatial variability of infiltration-related physical properties. The method of determining equation parameters should be appropriate for routine field use, and the parameters should be sufficiently sensitive to represent significant variations in infiltration associated with soil differences in a watershed.

Three different infiltration equations were employed to predict infiltration in well-drained Typic Torrox soils on the island of Oahu, Hawaii. Simple equations for calculating hydraulic conductivity, $K(\theta)$, and diffusivity, $D(\theta)$, were derived, and the parameters in the derived equations were determined from field measurements of steady infiltration and redistribution. Subsequently the parameters S and A in Philip's 2-term equation were calculated from $K(\theta)$ and $D(\theta)$. In order to adequately characterize the hydrologic properties of the surface soil to which the infiltration process is especially sensitive, the calculated sorptivity-antecedent water content relation, $S(\theta_0)$, was adjusted by an in-situ measured S , which was obtained by the method of Talsma. For the Green-Ampt equation, the field-saturated hydraulic conductivity, K_s , was measured directly in the field. The wetting front potential, H_f , in the equation was calculated from a derived simple algebraic

equation determined from field-saturated porosity and redistribution measurements.

Assessment of the Philip and Green-Ampt equations by comparison with measured infiltration at seven experimental sites with 14 infiltration measurements showed that results from the Philip equation had an average percentage error of 17% in predicting cumulative infiltration; the Green-Ampt equation was good only for predicting infiltration in relatively dry soil.

The Talsma-Parlange equation, which requires $S(\theta_0)$ and K_s but does not require $K(\theta)$ and $D(\theta)$, appeared especially promising for routine field use. The spatial variability of the field-measured S was best described by a log-normal distribution as indicated by the Kolmogorov-Smirnov test. In order to simplify obtaining $S(\theta_0)$, a linear relation between S and θ_0 was assumed and approximated from the geometric mean of field-measured sorptivity and $S = 0$ at saturation. This linear approximation was tested on seven soil locations with 26 infiltration measurements using the Talsma-Parlange equation to predict infiltration. The results showed an average percentage error of 23% in predicting cumulative infiltration. While infiltration predictions based on $K(\theta)$ and $D(\theta)$ obtained from field redistribution data were more accurate than the simplified Talsma-Parlange prediction, the latter is well adapted for extensive field use in watersheds.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	iii
ABSTRACT	iv
LIST OF TABLES	ix
LIST OF ILLUSTRATIONS	x
 CHAPTER ONE INTRODUCTION	 1
THEORETICAL AND EMPIRICAL APPROACH FOR INFILTRATION PREDICTION	 1
Theoretical Approach	1
Empirical Approach	3
OBJECTIVES OF THIS STUDY	5
 CHAPTER TWO DETERMINATION OF HYDRAULIC CONDUCTIVITY AND DIFFUSIVITY BASED ON IN-SITU SOIL WATER REDISTRIBUTION MEASUREMENTS	 7
INTRODUCTION	7
THEORETICAL BACKGROUND	9
EXPERIMENTAL PROCEDURES	11
Evaluation of Equations (2.3) and (2.4)	13
Determination of θ at "Field Saturation" and Calculation of K and D	17
Comparison of Field Measured and Calculated Hydraulic Conductivity	19
Determination of Matching Factor for $K(\theta)$ and $D(\theta)$ and the Results	22
CONCLUSION	24

	Page
CHAPTER THREE INFILTRATION PREDICTION BASED ON IN-SITU SOIL WATER REDISTRIBUTION MEASUREMENTS . . .	25
INTRODUCTION	25
PROCEDURES	26
Characteristics of Experimental Soils	26
Determination of Sorptivity and Coefficient A in Philip's Equation	28
RESULTS AND DISCUSSION	29
Calculated and Measured Parameters for Philip's Equation	29
Predicted Infiltration	30
SUMMARY AND CONCLUSION	40
CHAPTER FOUR PREDICTION OF GREEN-AMPT WETTING FRONT POTENTIAL AND ASSOCIATED INFILTRATION FROM IN-SITU SOIL WATER REDISTRIBUTION MEASUREMENTS	42
INTRODUCTION	42
METHODS	44
Derivation of Equation for Calculating Wetting Front Potential	44
Experimental Procedures	46
Determination of the Parameters in Equation (4.9)	47
Calculation of Infiltration Using Equation (4.1) . .	48
RESULTS AND DISCUSSION	49
Calculation of Wetting Front Potential	49
Predicted Infiltration	52
CONCLUSIONS	48

	Page
CHAPTER FIVE APPLICATION OF FIELD-MEASURED SORPTIVITY FOR SIMPLIFIED INFILTRATION PREDICTION . . .	59
INTRODUCTION	59
DESCRIPTION OF TALSMAN-PARLANGE INFILTRATION EQUATION	61
PROCEDURES	65
Determination of Sorptivity	65
Statistical Analysis	66
Field Methods	67
RESULTS AND DISCUSSION	70
Variability of Sorptivity in a Large Area	70
Calculation of Sorptivity and Prediction of Infiltration	71
SUMMARY AND CONCLUSION	76
CHAPTER SIX OVERALL CONCLUSIONS	78
APPENDIX I THE DETAILED DERIVATION OF EQUATIONS (2.1), (2.2), (2.7) and (2.8)	82
APPENDIX II PROOF OF IDENTITY OF MATCHING FACTORS FOR K AND D	86
APPENDIX III DERIVATION OF THE WETTING FRONT POTENTIAL EQUATION	87
APPENDIX IV EXAMPLE OF THE CALCULATION FOR THE DETERMINATION OF THE FIELD MEASURED SORPTIVITY DISTRIBUTION USING KOLMOGOROV-SMIRNOV TEST	90
APPENDIX V EXAMPLE OF SOLVING GREEN-AMPT EQUATION	91
LITERATURE CITED	94

LIST OF TABLES

Table		Page
2.1	Parameters for Water Redistribution Equations (2.3) and (2.4) Determined by Regression with Experimental Data from Seven Sites	14
2.2	Comparison of 85% Total Porosity with Measured "Field-Saturated" Soil Water Content	18
2.3	Comparison of Measured (K_s) and Calculated (K_c) Hydraulic Conductivity at Field Saturation	21
3.1	Calculated S and A for Combined Ap1 and Ap2 at Water Contents Corresponding to Measured S of Ap1 Layer for Seven Field Sites	33
3.2	Parameters Used in Calculating the Infiltration Rate for HSPA Site A	36
4.1	Parameters for Water Redistribution Equations (4.4) and (4.5) Determined by Regression with Experimental Data from Seven Sites, and the Calculated Wetting Front Potential from Equation (4.11)	50
4.2	Parameters for Equation (4.1) and the Comparisons of Calculated and Measured Results for All Seven Sites	54
5.1	The Results of Kolmogorov-Smirnov Test on the Field-Measured Sorptivity Distribution	72
AV.1	I versus RHS	92

LIST OF ILLUSTRATIONS

Figure		Page
2.1	Comparison of Calculated θ (from (2.3)).	15
2.2	Comparison of Calculated ψ (from (2.4)) with Measured Values for HSPA Site A.	16
2.3	Calculated Hydraulic Conductivity (cm/min) (semi-log scale) for HSPA Site A	20
2.4	Comparison of Calculated and Measured Hydraulic Conductivity and Diffusivity for HSPA Site A . . .	23
3.1	Illustration of Oxisol Soil Profile Infiltration Study. Molokai Silty Clay Loam, HSPA Site A . . .	27
3.2	Calculated and Measured Sorptivity As a Function of Water Content. Data Points Represent Direct Measurements of Sorptivity of Surface Soil. Molokai Soil, HSPA Site A.	31
3.3	Coefficient A of Philip's Infiltration Equation As a Function of Antecedent Soil Water Content. Molokai Soil, HSPA Site A.	32
3.4	Calculated Infiltration Rate at Different Antecedent Soil Water Contents (by vol.). HSPA Site A.	35
3.5	Comparison of Measured Infiltration Rate with Rate Curves Calculated by the Philip Equation with Both Matched and Unmatched Sorptivity. Molokai Soil, HSPA Site A, $\theta_0 = 28\%$ (by vol.) (dry run).	37
3.6	Comparison of Measured Infiltration Rate with Rate Curves Calculated by the Philip Equation with Both Matched and Unmatched Sorptivity. Molokai Soil, HSPA Site A, $\theta_0 = 33\%$ (by vol.) (wet run).	38

Figure		Page
3.7	Comparison of Calculated (Matched S) and Measured Cumulative Infiltration for Both Dry and Wet Antecedent Conditions at Seven Field Sites	39
4.1	The Relationships of Hydraulic Conductivity and Wetting Front Potential; HSPA Site A	51
4.2	Comparison of Measured and Calculated (by Green-Ampt Approach) Infiltration in Dry Soils	55
4.3	Comparison of Calculated and Measured Cumulative Infiltration for Molokai Soil, HSPA Site A	57
5.1	Experimental Layout for Infiltration and Sorptivity Measurements	68
5.2	Sorptivity as a Function of Antecedent Water Content--A Linear Approximation for HSPA, Site A	73
5.3	Comparison of Calculated and Measured Cumulative Infiltration.	75
AV.1	Illustration of Solving Cumulative Infiltration, I, in Green-Ampt Equation for HSPA Site A Dry Infiltration Run	93

CHAPTER ONE

INTRODUCTION

In watershed simulation, one of the most important processes to be considered is infiltration. The infiltration process divides precipitation or other surface water between overland flow and subsurface flow. Therefore, it can affect not only the timing, but also the distribution and magnitude of surface runoff.

Infiltration rate, i.e. the instantaneous flux of water through the soil surface, is highly dependent upon the condition of the soil surface. After water has infiltrated the soil surface, the rate of downward movement is controlled by the characteristics of the soil and also by water content in the profile. The process of infiltration is very complex and only partially understood [Hjelmfelt and Cassidy, 1975]. From the hydrological point of view, infiltration is complicated both by a highly variable supply of water to the infiltrating surface and soil characteristics that vary in both time and space [Fleming and Smiles, 1973]. Therefore, every soil-cover-moisture complex will have different rainfall-related infiltration characteristics. Since infiltration is an important component of the rainfall-runoff process in a watershed, a reliable estimation of infiltration is required for any watershed model.

THEORETICAL AND EMPIRICAL APPROACHES FOR INFILTRATION PREDICTION

Theoretical Approach

Infiltration theory is one aspect of the theory of fluid flow through porous media. It was developed from experimental data for

non-swelling materials, and the theory now has been applied to cases of flow through fairly complicated media [Fleming and Smiles, 1973]. In this study, we will examine the various approaches that have been used in attempts to characterize the infiltration process mathematically; little attention will be given to the derivation of infiltration theory nor the history of theory development.

The general flow equation describing the infiltration process is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[K \frac{\partial \psi}{\partial z} \right] + \frac{\partial K}{\partial z} \quad (1.1)$$

in which

- θ is soil water content, cm^3/cm^3 .
- ψ is soil water potential, cm of water.
- z is the depth of soil profile, cm.
- t is the time, min.
- K is hydraulic conductivity, cm/min.

Equation (1.1) is a combination of Darcy's Law with the continuity equation [Nielsen et al., 1972]. It is a nonlinear, second-order partial differential equation, which was derived with the assumption that the soil matrix is rigid, isothermal and isotropic. The detailed assumptions behind (1.1) are given by Klute [1973].

In order to solve (1.1), we must know the hydraulic conductivity-water content, $K(\theta)$, and diffusivity-water content, $D(\theta)$, functions that characterize the soil. Also, the initial and boundary conditions of the equation that describe the flow equation need to be defined [Klute, 1973].

Equation (1.1) can be solved either analytically [e.g. Philip, 1969; Talsma and Parlange, 1972] or numerically (e.g. Klute, 1952; Hank and Bowers, 1962]. Recently, (1.1) has also been solved using a perturbation method [Babu, 1976; Liu, 1976] and a finite element method [Remson et al.,

1971; Cheng, 1975b]. However, as concluded by Klute [1973], field soils do not often conform to the assumptions associated with equation (1.1). Additionally, in a watershed it is extremely difficult to characterize the soil-water-related properties of an entire watershed in detail due to limitations in measuring techniques. The major problems of applying (1.1) are still (a) how to obtain reliable $K(\theta)$ and $D(\theta)$ measurements of the soil, and (b) how to define the initial and boundary conditions of flow. (The problems of measuring $K(\theta)$ and $D(\theta)$ will be discussed in Chapter Two.) Therefore, the detailed theoretical approach of solving the watershed infiltration problem using solutions to (1.1) is not recommended.

Empirical Approach

There are a number of equations which have originated from the analysis of experimental data, for example the Kostiaikov equation [1932]. In order to apply the equation, it is necessary to determine the parameters of the equation from the experimental data first. Some of the simple algebraic infiltration equations are derived from physical infiltration processes, such as the Green-Ampt equation [1911], the Horton equation [1940], the Holtan equation [1961] and the Collis-George equation [1977]. Furthermore, some algebraic infiltration equations are analytically derived from (1.1) with certain assumptions, such as the Philip two-parameter equation [1957] and the Talsma and Parlange equation [1972]. These simple algebraic equations are of considerable current interest because of their simplicity and accuracy. There are some other advantages of using algebraic equations; the parameters involved can be adjusted to account for complexities which have been

eliminated in mathematical analysis to render the problem soluble [Baver et al., 1972]. Because most empirical equations are expressed as a function of time and total quantity of water infiltrated into the soil, they are relatively convenient for use in runoff studies [Cheng, 1975a].

However, in order to apply a simple algebraic infiltration equation, the parameters need to be known first. Some of the parameters involved in certain equations are physically based. The parameters may or may not be measured directly on the soil. On the other hand, some of the parameters which are not physically based, but are useful in the equation, must be regressed from experimental data. Usually, the regressed parameters are independent of initial and boundary conditions. Hence, very often the regressed parameters can only be applied to a certain set of data, but are not applicable to the general case. This implies that most of the non-physically based parameters (or equations) may not be able to handle temporal and spatial variability of infiltration in a watershed.

Therefore, in watershed infiltration analysis, temporal and spatial variability of rainfall and the infiltration-related soil conditions make the application of soil physics difficult. The variability of infiltration parameters will affect the amount of infiltrated water. Hence, it is believed that detailed prediction of infiltration in a watershed will depend upon the prediction of infiltration-related parameters of the space and time locations. A simple field method is needed which can be used at a large number of sites in a watershed to characterize variability of infiltration-related physical properties.

OBJECTIVES OF THIS STUDY

The overall goal of this research is prediction of water infiltration in field soils for the range of antecedent conditions common to watersheds. This requires the development and field testing of methods appropriate for common field use; the approach is specified by four related objectives:

1. modify the simple field method of Nielsen et al. [1973] so that the hydraulic conductivity, $K(\theta)$, and diffusivity, $D(\theta)$, can be obtained over a wider range of soil-water contents;
2. utilize field-measured conductivity and diffusivity data to calculate sorptivity-water content relations, $S(\theta_0)$, which can be used in combination with direct field sorptivity measurements to predict infiltration for given antecedent water contents;
3. derive a simple algebraic equation, based on in-situ soil water redistribution measurements, for estimating the wetting-front potential in the Green-Ampt equation, which will be used for predicting infiltration; and
4. find a simple algebraic infiltration equation for which the parameters are field-measured and capable of accommodating spatial variability and changes in antecedent water contents.

Field measurements were conducted on three field locations on the island of Oahu, Hawaii. The soils are highly aggregated and well-drained Torrox soils of the Molokai and Lahaina soil series. The intent of

having three soil locations was to provide replication in testing the proposed methods rather than for a comparison of soils.

The dissertation is divided into six chapters, the first being the Introduction; Chapters Two, Three, Four and Five address Objectives 1, 2, 3 and 4, respectively; Chapter Six provides the overall conclusions. Some detailed derivations of equations used in the study are contained in the Appendices, along with some data and sample calculations.

CHAPTER TWO

DETERMINATION OF HYDRAULIC CONDUCTIVITY AND DIFFUSIVITY BASED
ON IN-SITU SOIL WATER REDISTRIBUTION MEASUREMENTS

INTRODUCTION

The variation of soil physical properties is difficult to measure or to reproduce accurately either by material or symbolic modeling. It is often difficult, therefore, to determine meaningful parameters for watershed infiltration simulation. Thus, even though the theory of infiltration is well understood [Klute, 1973], application of the theory to the field situation is questionable unless more relevant soil parameters can be determined.

Redistribution of soil water is an extension of the infiltration process, and this continuous process will influence the next cycle of infiltration. In order to simulate infiltration in watershed modeling, therefore, one must identify, in both time and space, the antecedent soil conditions.

In order to simplify the problem, it is often assumed that the theory of soil water flow can accommodate the vertical changes in soil physical properties, but spatial variability in the horizontal dimensions must be characterized. Recently, the distributions of soil physical properties measured in the field for soil water movement prediction have been studied by several researchers including Rogowski [1972], Nielsen et al. [1973], Biggar et al. [1976], Warrick et al. [1977] and

Peck et al. [1977]. Since water conducting and water storage properties of soils within a watershed vary from site to site within the same soil series or between different soil series, objective techniques are needed to adequately characterize these properties; such techniques must be sufficiently simple and economical to provide the necessary number of measurements to adequately characterize the entire watershed.

Determination of hydraulic conductivity as a function of soil water content is tedious and time consuming [Klute, 1973]. The methods of measuring hydraulic conductivity and soil water characteristics from soil cores have been described in detail; for example see Nielsen et al. [1972]. Methods of calculating hydraulic conductivity from the soil-water characteristic, introduced by Childs and Collis-George [1950], have been modified and tested by a number of investigators; Gardner [1974] has discouraged the general use of such methods. The capillary theory upon which these methods are based is not always appropriate to field soils, and, additionally, soil cores often do not constitute a representative elementary volume [Bear, 1972].

Field measurement of hydraulic conductivity and soil water characteristics has been discussed by Nielsen et al. [1972] and Klute [1973]. The detailed Darcian analysis of redistribution data [Nielsen et al., 1973; Ahuja et al., 1975] is laborious and expensive, and also involves fairly complicated mathematical calculations. Detailed field measurement, therefore, will not be conducted on a routine basis, hence a simple field method is needed.

One simplified field method for measuring hydraulic conductivity and diffusivity was developed by Nielsen et al. [1973] by assuming a

unit hydraulic gradient in the soil profile during the redistribution period. On the average, the results obtained by this simple field method compare favorably with the detailed Darcy analysis. A limitation of the simple field method of Nielsen et al. is that hydraulic conductivity and diffusivity are determined only within the measured range of soil water contents. Extension of the simplified field method to provide a characterization of water conducting and water storage properties of soils over a wider range of water contents requires further improvement of the method. The objective of the study reported in this chapter is to modify the simple field method of Nielsen et al. [1973] so that the hydraulic conductivity and diffusivity can be obtained over a wider range of soil-water contents.

THEORETICAL BACKGROUND

The assumption of a unit hydraulic gradient, for water redistribution after steady infiltration in a uniform soil profile without evaporation, was introduced by Black et al. [1969]. With this assumption, the rate of change of soil water content in the profile can be used to calculate hydraulic conductivity, $K(\theta)$, as shown by Nielsen et al. [1973]:

$$K_L = -L \frac{d\theta}{dt} \quad (2.1)$$

where

- L is depth of soil profile, cm.
- K_L is hydraulic conductivity at depth L , cm/min.
- θ_L is average soil water content in soil profile, cm^3/cm^3 .
- t is time, min.

Furthermore, following Nielsen et al. [1973], if we assume that an average soil water characteristic curve holds for the entire soil profile or soil layer under consideration, along with the assumption of unit gradient, then the diffusivity of the soil profile, at depth L , can be expressed as,

$$D_L = -L \frac{d\psi}{dt} \quad (2.2)$$

where

D_L is soil water diffusivity at depth L , cm^2/min .
 ψ_L is total soil water potential at L , cm of water.

Extension of this simplified method to allow calculation of K and D at water contents higher or lower than those measured during drainage requires that we develop mathematical expressions which adequately describe θ and ψ versus time during drainage. Following Richards et al. [1956] and Gardner et al. [1970], we assume that water content in the soil profile during the post-infiltration redistribution process has a log-linear relationship with time, such that,

$$\theta = at^b \quad (2.3)$$

where

a and b are constants.

In this study, it is also assumed that the total soil water potential during the redistribution period likewise can be expressed as a power function of time, that is,

$$\psi = mt^n \quad (2.4)$$

where

m and n are constants.

Substituting (2.3) and (2.4) into (2.1) and (2.2), respectively, we obtain (2.5) and (2.6) in which K and D are both expressed as functions of t.

$$K_L = -L abt^{b-1} \quad (2.5)$$

$$D_L = -L mnt^{n-1} \quad (2.6)$$

Furthermore, if we substitute (2.3) into (2.5) and (2.6), K and D can be expressed in terms of θ , such that (2.5) and (2.6) become,

$$K_L = -L ba \left(\frac{1}{b}\right) (\theta) \left(\frac{b-1}{b}\right) \quad (2.7)$$

$$D_L = -L mna \left(\frac{n-1}{b}\right) (\theta) \left(\frac{n-1}{b}\right) \quad (2.8)$$

Thus, hydraulic conductivity and diffusivity of the soil profile at depth L can be calculated with (2.7) and (2.8) for all soil water contents, if the constants a, b, m and n can be determined. The reliability of (2.3) and (2.4) for describing soil water behavior in the field will be verified by the experimental results. The detailed derivations of (2.1), (2.2), (2.7) and (2.8) are available in Appendix I.

EXPERIMENTAL PROCEDURES

In order to test the use of (2.7) and (2.8), field experiments were conducted on soils of the Molokai and Lahaina series on the Wahiawa Plateau of the island of Oahu. Detailed descriptions of the soils are given by Green et al. [1979]. All test sites were in sugarcane fields with tilled Ap horizons 30 to 40 cm deep. Site preparation involved

leveling of the soil surface followed by shallow hoeing and a final leveling.

Infiltration measurements were conducted with a double-ring infiltrometer in a manner similar to that described by Ahuja et al. [1975], but with only a 2-cm head maintained by controlling water flow to the inner and outer rings. Cumulative infiltration over time was measured in the 30-cm diameter inner ring while a buffer zone was provided by the 120-cm outer ring. The rings were inserted 15 to 20 cm into the soil. Initial wetting of the profile was accomplished with an infiltration run on dry soil. After a redistribution period of one day, a multiple tensiometer was installed at the center of the inner ring. The porous cups of the multiple tensiometer were located at 10 and 20 cm from the ground surface in most measurements, but in some installations the cup depths were 7.6 and 22.9 cm.

After tensiometer installation, redistribution of soil water was allowed to proceed for another two days, followed by the infiltration run on moist soil. Water application was continued approximately one hour beyond the time that an apparent steady infiltration rate was observed.

After the water supply was cut off and when the water in the ring had just disappeared from the ground surface, time t was set equal to zero for the starting time of redistribution. The soil water potential data during the redistribution period were obtained from the multiple tensiometer readings. The soil water content data were obtained gravimetrically from soil samples obtained between the inner and the outer ring; duplicate samples were taken to depth L at each sampling time.

Volumetric water contents were calculated later using bulk density and particle density data obtained from soil cores taken from the site after redistribution measurements were terminated (usually after 10 days).

The soil surface inside the rings was covered with a plastic sheet during redistribution to prevent evaporation. A 2-cm thick styrofoam sheet was placed on the plastic sheet to reduce extreme changes in soil temperature. A canopy was installed above the experimental setup to prevent rainfall from entering the rings.

Evaluation of Equations (2.3) and (2.4)

Infiltration and redistribution measurements were made on two to three experimental sites, 10-20 meters apart, at each of three different soil locations, giving a total of seven experimental sites. Results from the regression of (2.3) and (2.4) on experimental data for each plot are tabulated in Table 2.1. Examples of experimental and regressed results are shown also in Figures 2.1 and 2.2. The correlation coefficient, r , between regressed and measured results for θ versus t exceeded -0.95 for all plots, and the standard deviation of the residual, s_θ , is about 1% of soil water content by volume. For ψ versus t , r is always larger than -0.98, and s_ψ is less than 1.2 cm of water. Practically speaking, on the basis of the results shown in this study, Equations (2.3) and (2.4) are very good empirical equations for describing changes of θ and ψ versus time during post-infiltration redistribution of water in the soil profile.

Table 2.1 Parameters for Water Redistribution Equations (2.3) and (2.4) Determined by Regression with Experimental Data from Seven Sites.

Location		Equation							
		$\theta = at^b$				$\psi = mt^n$			
		a	b	r	s_θ	m	n	r	s_ψ
HSPA	A	0.6079	-0.0595	-0.9949	0.101	-8.5570	0.3259	-0.9986	1.026
	B	0.6602	-0.0633	-0.9928	0.102	-1.1095	0.5505	-0.9903	1.192
	C	0.6071	-0.0611	-0.9913	0.102	-4.8220	0.3807	-0.9967	1.040
OP221	E	0.5895	-0.0601	-0.9874	0.102	6.6110	0.3555	-0.9990	1.024
	W	0.7132	-0.0769	-0.9965	0.101	-8.1103	0.3446	-0.9959	1.073
OP410	E	0.7058	-0.0797	-0.9871	0.101	-5.8426	0.3718	-0.9996	1.011
	W	0.6110	-0.0601	-0.9944	0.101	-12.1452	0.2761	-0.9990	1.026

r is the correlation coefficient.

s_θ and s_ψ are standard deviations of the residuals of θ and ψ , respectively.

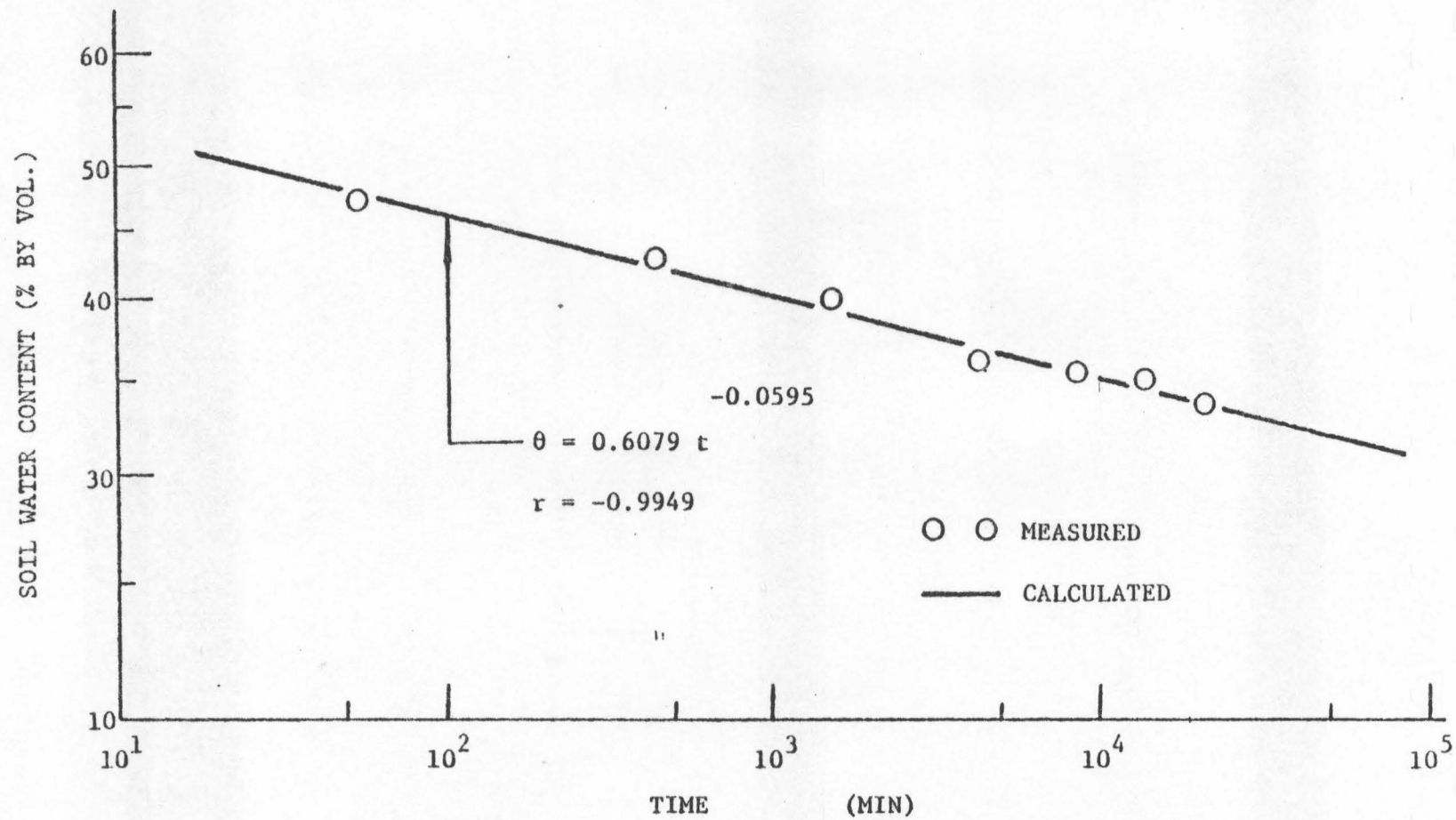


Figure 2.1 Comparison of Calculated θ [from (2.3)] with Measured Values for HSPA Site A.

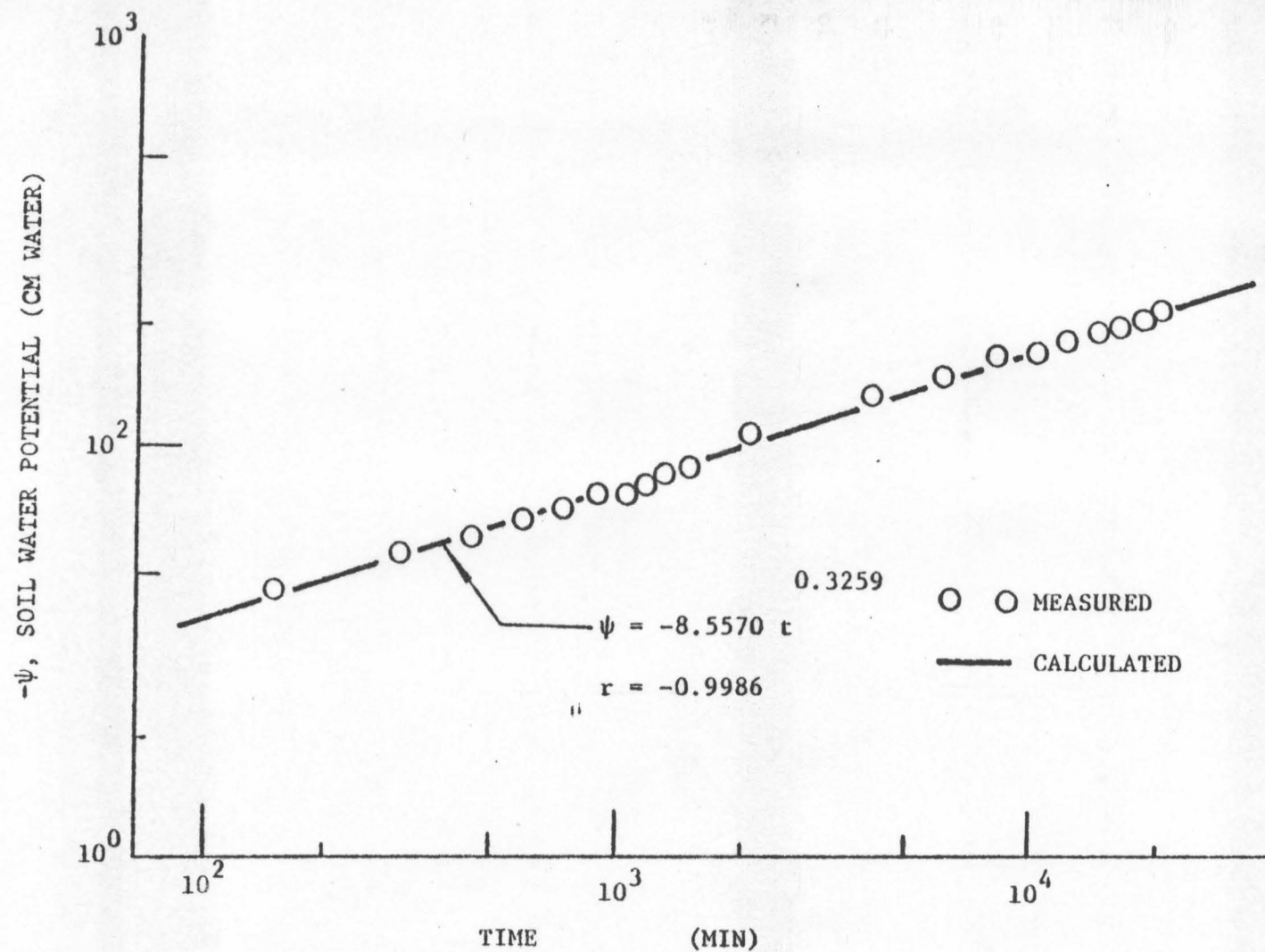


Figure 2.2 Comparison of Calculated ψ [from (2.4)] with Measured Values for HSPA Site A.

Determination of θ at "Field Saturation" and Calculation of K and D

Hydraulic conductivity and diffusivity of the soil profile can be calculated either from Equations (2.5) and (2.6) or from (2.7) and (2.8). No matter which set of equations is used for calculating K and D, the water content θ or the appropriate t at "field saturation" has to be determined.

If (2.5) and (2.6) are going to be applied for calculating K and D, then t should be equal to zero for "field saturation" at the beginning of the redistribution process. Unfortunately, when t is equal to zero, K in (2.5) is undefined because b always has a negative value. An arbitrary choice of a small value of t to satisfy (2.5) near zero time was considered unsatisfactory.

On the other hand, if (2.7) and (2.8) are used, θ instead of t has to be determined. At the fully saturated condition, θ should be equal to total porosity of the soil. However, reports from numerous studies state that even though a soil is submerged in water, the soil is not fully saturated due to air entrapment. For example, Jackson [1963] found that for loams only 79 to 91 percent of total porosity was fillable by water. In a previous field study at a site near our HSPA site [Green, Rao and Balasubramanian, 1972, unpublished data], soil water contents were measured by neutron probe at the time when the ponded water had just disappeared from the ground surface. The results in column 5 of Table 2.2 show saturation percentages of 75 to 86% with a median value of about 85%; this saturation percentage is very close to the average value obtained by Jackson. Hence, 85% of total porosity is used as the "field saturated" soil water content in this study.

Table 2.2 Comparison of 85% Total Porosity with Measured "Field-Saturated"
Soil Water Content [Unpublished data from
Green, Rao and Balasubramanian].

(1) Depth (cm)	(2) Total Porosity (% by Vol)	(3) 85% Total Porosity (% by Vol)	(4) Measured "Field Saturated" θ (% by Vol)	(5) Saturation % (4)/(2) (%)
P L O T N O . 2				
20	61.2	52.0	45.7	74.7
40	54.2	46.1	46.0	84.9
60	51.7	43.9	44.4	85.9
80	54.1	46.0	45.7	84.5

P L O T N O . 4				
20	54.9	46.7	45.2	82.3
40	57.1	48.5	49.2	86.2

Since θ in (2.7) and (2.8) at "field saturation" can be determined, K and D at any level of soil water content of interest can be calculated by the equations. An example of calculated K versus θ , plotted on a semi-log scale, is shown in Figure 2.3. The calculated D - θ curve (not shown) has about the same shape.

Comparison of Field Measured and Calculated Hydraulic Conductivity

During the infiltration measurement when an apparent steady infiltration rate was observed, the flux in the inner ring and the water potential readings from the multiple tensiometer were recorded. Conventionally it is assumed that the hydraulic conductivity is equal to the "steady" field measured flux; in other words, a unit hydraulic gradient along the soil profile is assumed. But, for the field soils of this study, the soil profile is not uniform with depth; most of the soils have about 40 cm of plow layer, below which is a relatively dense B horizon with greater impedance to water flow than the plow layer. It is likely that the hydraulic gradients less than 1.0 shown in Table 2.3 are the result of flow impedance in the B horizon at "field saturation." The actual hydraulic conductivity, therefore, should equal the measured flux divided by the measured hydraulic gradient. A comparison of field measured hydraulic conductivity and hydraulic conductivity calculated at the water content corresponding to 85% total porosity for all experimental plots is given by data tabulated in Table 2.3.

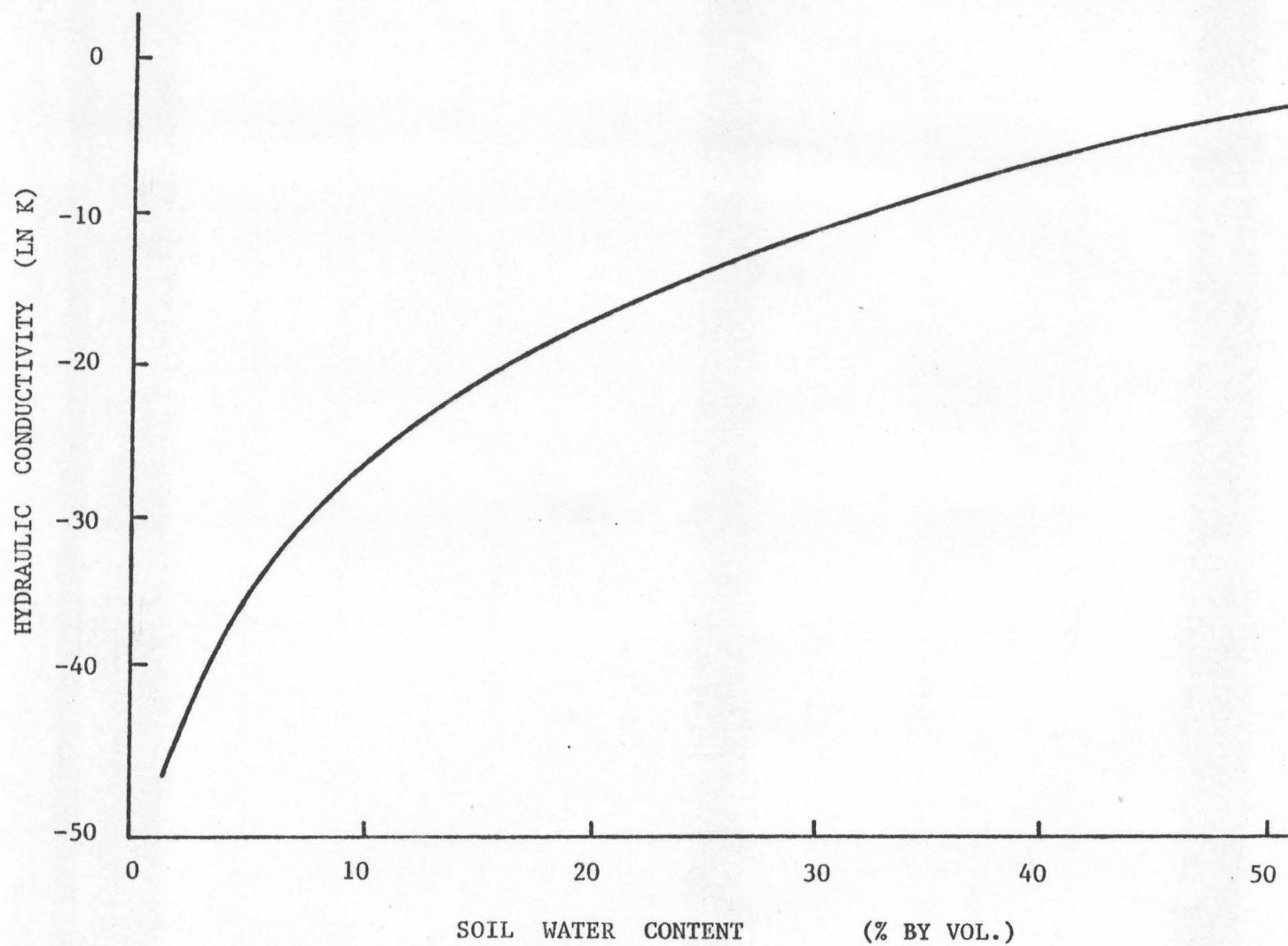


Figure 2.3 Calculated Hydraulic Conductivity (cm/min)
(semi-log scale) for HSPA Site A.

Table 2.3 Comparison of Measured (K_s) and Calculated (K_c) Hydraulic Conductivity at Field Saturation

Plot No.	Gradient ($\Delta\psi/\Delta z$)*	Flux i_s cm/min	K_s cm/min	K_c cm/min	Matching Factor K_s / K_c
HSPA A	0.90	0.0370 (258)	0.0411	0.0259	1.587
B	1.00**	0.0217 (282)	0.0217	0.0151	1.437
C	1.00	0.0083 (229)	0.0083	0.0135	0.615
OP221 E	0.72	0.1050 (160)	0.1454	0.1667	0.872
W	0.50	0.0505 (195)	0.1010	0.0208	4.856
OP410 E	0.66	0.0612 (273)	0.0933	0.0382	2.442
W	0.70	0.0876 (140)	0.1251	0.1108	1.129

* Field measured at "steady" condition.

** Assumed.

K_c Calculated hydraulic conductivity from Equation (2.7) at 85% total porosity.

i_s Field measured flux at the time (in minutes) denoted in the parentheses.

K_s Hydraulic conductivity calculated from $K_s = i_s / (\Delta\psi/\Delta z)$.

Determination of Matching Factor for $K(\theta)$ and $D(\theta)$ and the Results

From Table 2.3 we see that the calculated values are reasonably close to the measured results for most cases. For practical purposes it is expedient to adjust the calculated $K(\theta)$ curve such that it passes through the measured field-saturated conductivity value; this can be done (although without theoretical rationale) by shifting the entire curve on the vertical axis by multiplication with a matching factor. A similar use of a matching factor, defined as the ratio of measured conductivity value to the calculated value, was introduced by Jackson et al. [1965] and has been used by Kunze et al. [1968] and Green and Corey [1971] in calculating the hydraulic conductivity from the soil water characteristic curve. The assumptions of the matching technique are that the measured result is the true value for describing the soil characteristics of interest and also, at all points within the range of interest, the ratio between measured and calculated values is constant. This implies that the matching factor can be obtained for any water content at which the hydraulic conductivity can be measured; saturation is probably most common. But, for diffusivity it is not easy to obtain a field measured value. It can be shown that the matching factor for diffusivity is identical to the matching factor for hydraulic conductivity; the detailed derivation of this relation is shown in Appendix II. The matching factors for all experimental plots are shown in the last column of Table 2.3. Examples of matched $K(\theta)$ and $D(\theta)$ are shown in Figure 2.4. Such curves, which are admittedly approximate, might be expected to represent water movement in field soils with sufficient accuracy for many applications in watershed hydrology and irrigation.

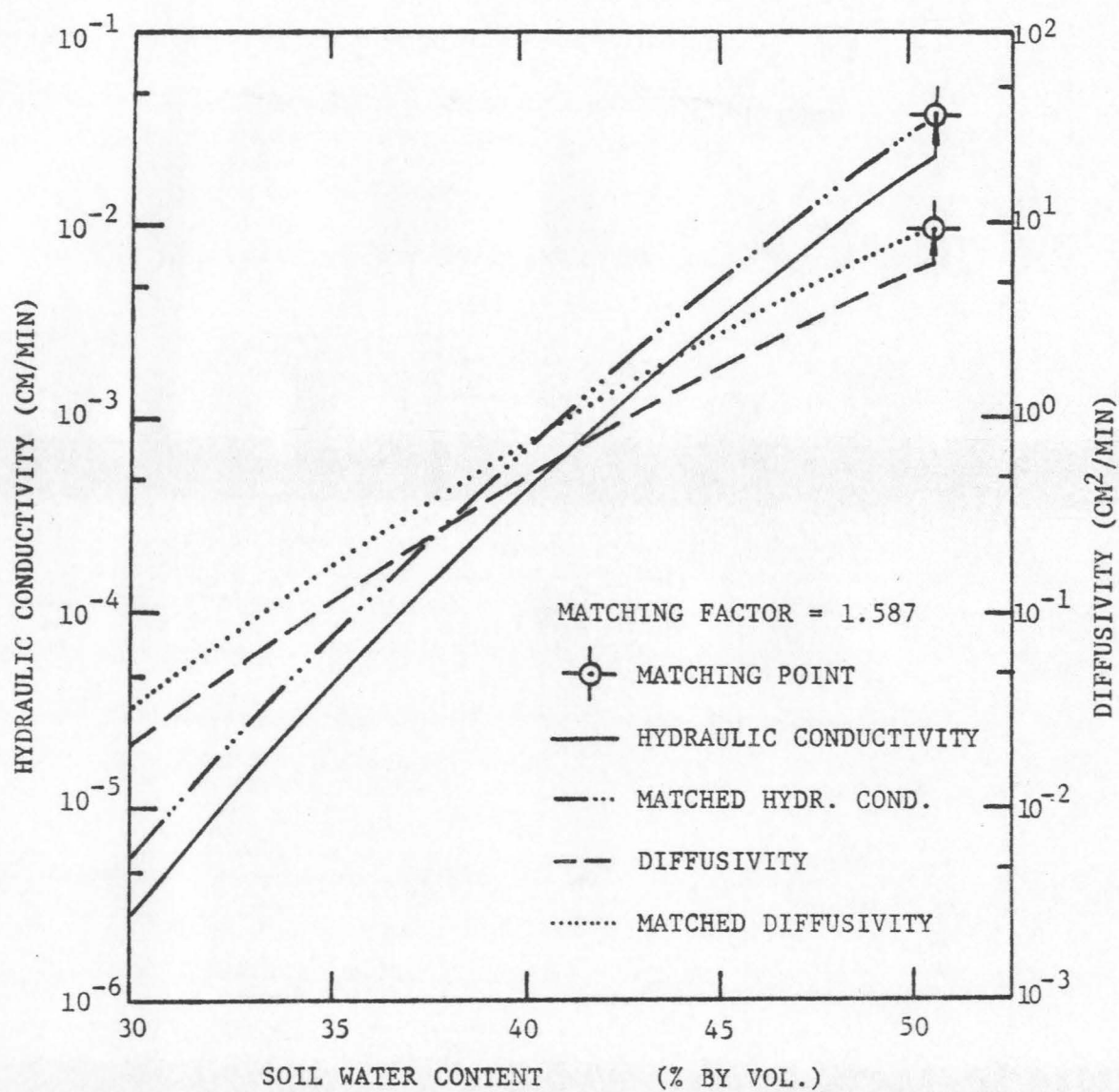


Figure 2.4 Comparison of Calculated and Matched Hydraulic Conductivity and Diffusivity for HSPA Site A.

CONCLUSIONS

Several conclusions can be drawn from the proposed method of measuring hydraulic conductivity and diffusivity.

1. The proposed method is comparatively simple and rapid. Usually the field measurement can be completed within ten days after redistribution has started.
2. The calculations of $K(\theta)$ and $D(\theta)$ are simple and can be obtained for a wide range of soil water contents.
3. $K(\theta)$ and $D(\theta)$ are measured from a transient-state flow system in the field. They should be relevant to a description of the actual soil-water movement in the field.
4. The assumption of 85% of total porosity as a "field saturated" condition seems to be reasonable. However, more experimental data are needed in order to strengthen this conclusion.
5. The matching factors for $K(\theta)$ and $D(\theta)$ are identical; this can be proved mathematically by the definition of $D(\theta)$ combined with Equations (2.1) and (2.2).

The application of calculated $K(\theta)$ and $D(\theta)$ for infiltration prediction will be evaluated and discussed in Chapter Three.

CHAPTER THREE
INFILTRATION PREDICTION BASED ON IN-SITU SOIL WATER
REDISTRIBUTION MEASUREMENTS

INTRODUCTION

In watershed simulation, characterization of infiltration is complicated by temporal and spatial variations in both antecedent soil water contents and surface conditions. An infiltration equation is needed, therefore, that will accommodate a range of antecedent water contents and account for variability in water conduction and storage properties of the soil. In this study, Philip's two-parameter equation [Philip, 1957] will be used to predict infiltration for various antecedent water contents. The equation is,

$$I = St^{\frac{1}{2}} + At \quad (3.1)$$

in which

I is cumulative infiltration, cm.
S is sorptivity, $\text{cm}/\text{min}^{\frac{1}{2}}$.
A is a coefficient, cm/min .
t is time, min.

Philip's equation is physically based and was derived from water flow theory. Thus, the parameters S and A in the equation are actually functions of antecedent water content, and are functionally related to the hydraulic conductivity-water content, $K(\theta)$, and diffusivity-water content, $D(\theta)$, functions which, in our work, have been determined from measured field infiltration and drainage data. In this paper, we will use the matched $K(\theta)$ and $D(\theta)$ obtained in Chapter Two to calculate S

and A values appropriate for a given soil at the existing antecedent water content. These calculated infiltration parameters are subsequently used in the Philip equation to obtain infiltration rate over time. Additionally, an independent, rapid, direct method of measuring S at the soil surface [Talsma, 1969] provides a means of characterizing sorptivity at the immediate soil surface, in contrast to the calculated S which characterizes the entire plow layer. The calculation method of Philip and direct measurement method of Talsma are combined in this study in an attempt to develop a practical means of predicting infiltration in a watershed. Predicted infiltration is compared with field-measured infiltration for several well-drained Hawaii Typic Torrox soils for various antecedent water contents.

PROCEDURES

Characteristics of Experimental Soils

In this study, the experimental soils are the agricultural soils specified in Chapter Two. A typical profile of these soils is shown diagrammatically in Figure 3.1. Of major interest is the Ap horizon, a plow layer 30 to 40 cm deep. Below the Ap horizon is a relatively dense, undisturbed B horizon. Due to an increase in bulk density with depth in the plow layer, the Ap horizon is divided into two layers, designated Ap1 and Ap2. The Ap1 or surface layer, is usually less than 10 cm thick. Since the bulk densities of these layers differ, it is likely that the associated pore-size distributions are also different; this suggests that $K(\theta)$ and $D(\theta)$ calculated from the simplified method in Chapter Two may not adequately describe the characteristics of the

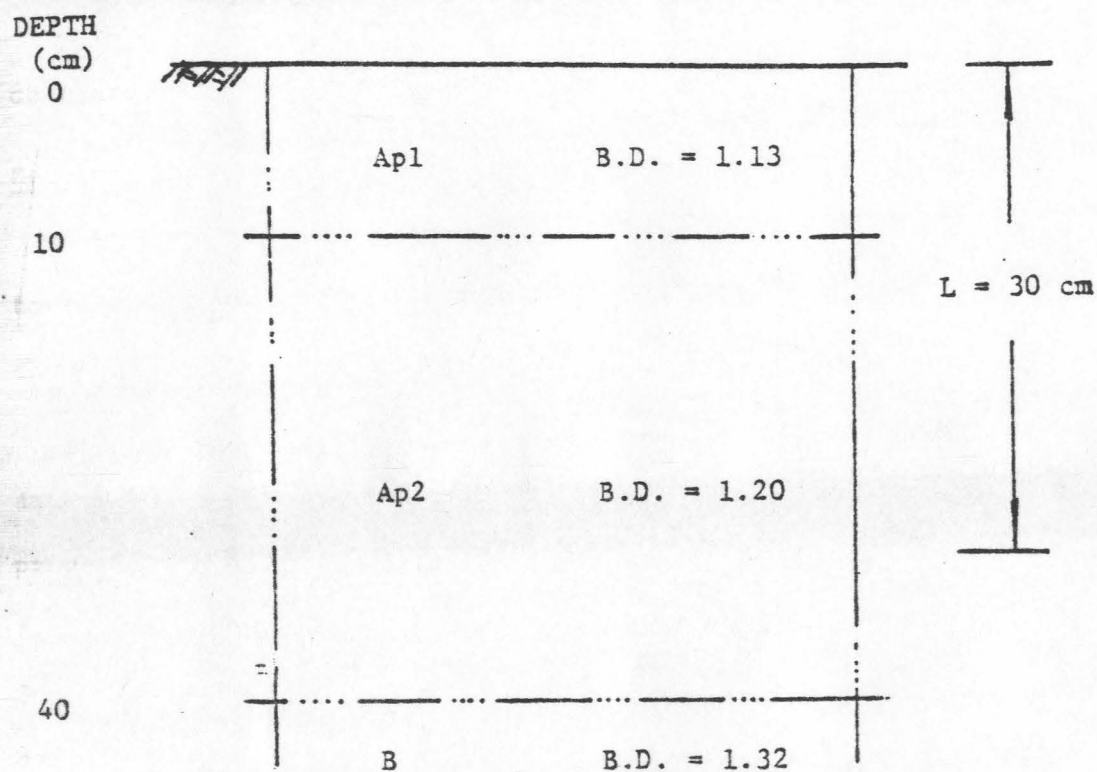


Figure 3.1 Illustration of Oxisol Soil Profile Infiltration Study. Molokai Silty Clay Loam, HSPA Site A.

soil in the A_{pl} layer. Also, it is likely that the properties of the A_{pl} layer will control the infiltration process during the early times after ponding. These considerations suggest a separate sorptivity characterization of the A_{pl}.

Determination of Sorptivity and Coefficient A in Philip's Equation

In order to use (3.1) for predicting infiltration, it is necessary to find the two parameters, the sorptivity and the coefficient A, in the equation first.

S and A in Philip's equation can be calculated as a function of antecedent soil water content, θ_0 , based on the known soil physical properties $K(\theta)$ and $D(\theta)$. The methods of calculating $S(\theta_0)$ and $A(\theta_0)$ from $K(\theta)$ and $D(\theta)$ have been developed by Philip [1955]. In this study, S and A are calculated from the matched $K(\theta)$ and $D(\theta)$ following the calculation procedures outlined by Kirkham and Powers [1972].

Sorptivity is the most important single quantity governing the early portion of infiltration as represented by Philip's equation [1957]. This implies that for the early portion of infiltration $S(\theta_0)$ is determined primarily by the pore-size distribution of the soil near the surface, viz. the A_{pl} layer in this study. The $A(\theta_0)$ term in the Philip equation, on the other hand, contributes more to calculated infiltration at later times, and is probably not so sensitive to changes in physical properties with depth as is $S(\theta_0)$. This suggests that the $K(\theta)$ and $D(\theta)$ data from Chapter Two, which are applicable to the entire A_p horizon, can be used to calculate appropriate $A(\theta_0)$ values for the entire A_p horizon at various antecedent water contents. However, the associated $S(\theta_0)$ calculated from $K(\theta)$ and $D(\theta)$, while appropriate for

most of the A_p , may not adequately characterize the surface layer and thus may require a separate assessment for the A_{pl} layer.

The simple field sorptivity measurement of Talsma [1969] was used to characterize the surface soil. The details of sorptivity measurement and an assessment of variability of sorptivity for the soils used in this study will be discussed in Chapter Five. Talsma's method is simple but the method provides only one value of S , that which corresponds to the antecedent water content at the time of measurement. In order to obtain sorptivity of the same soil at different antecedent water contents, we would have to make a series of sorptivity measurements at selected antecedent conditions. In the field it is extremely difficult to vary antecedent water contents for a series of S measurements, especially on unstable, tilled soils. One alternative is to adjust the calculated $S(\theta_0)$ by matching the curve to a single sorptivity point measured directly by the Talsma method in the surface soil layer. The concept and the way of using a matching factor have been discussed in Chapter Two, and will not be repeated here. The measured sorptivities of surface soil in a large area appear to be log-normally distributed [Brutsaert, 1976; also see Chapter Five]. Hence, the geometric mean of the field-measured sorptivities is used as a matching point and the matched $S(\theta_0)$ can be easily obtained.

RESULTS AND DISCUSSION

Calculated and Measured Parameters for Philip's Equation

Examples of calculated S and A from $K(\theta)$ and $D(\theta)$ of Chapter Two are shown as functions of antecedent soil water content in Figures 3.2

and 3.3. In Figure 3.2, the calculated S is shown by the dashed line. The circles are the field-measured sorptivities obtained by Talsma's method. The crossed point is the matching point, which is the geometric mean of the measured sorptivities (on the x-axis, the arithmetic mean of soil water content is used because antecedent soil water content is normally distributed [Chong and Green, 1979, unpublished data]). The solid curve is the matched sorptivity curve. In Figure 3.2, the field-measured sorptivities of surface soil are considerably higher than the calculated S . These results are consistent with the measured increase in bulk density with depth (Figure 2.1), assuming that for a given soil texture, sorptivity generally will be inversely related to bulk density. This relationship is suggested by the results of Bligh [1978]; subsoils had lower sorptivities than surface soils. Thus, it is reasonable to conclude that the calculated $S(\theta_0)$ curve (before matching) describes approximately the sorptivity of the Ap2 but not that of the Ap1 layer. A comparison of measured and calculated sorptivities at the appropriate soil water contents, together with the matching factors, are shown for all field sites in Table 3.1.

Figure 3.3 illustrates the variation of A as a function of antecedent soil water content. The calculated A for a given water content is used directly in Philip's equation to predict infiltration at that antecedent water content.

Predicted Infiltration

Since S and A as functions of antecedent soil water content have been obtained, the infiltration rate or the cumulative infiltration at

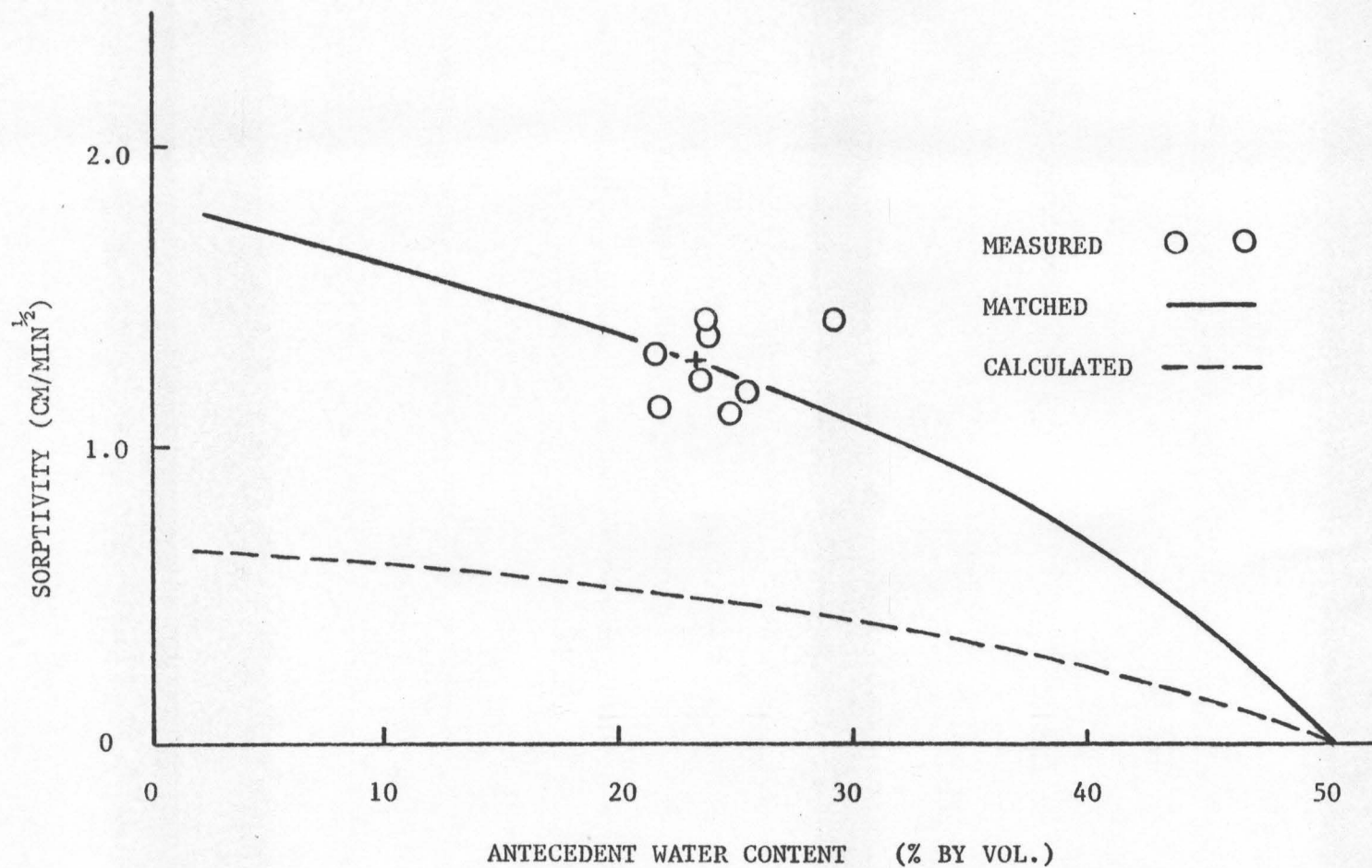


Figure 3.2 Calculated and Matched Sorptivity As a Function of Water Content. Data Points Represent Direct Measurements of Sorptivity of Surface Soil. Molokai Soil, HSPA Site A.

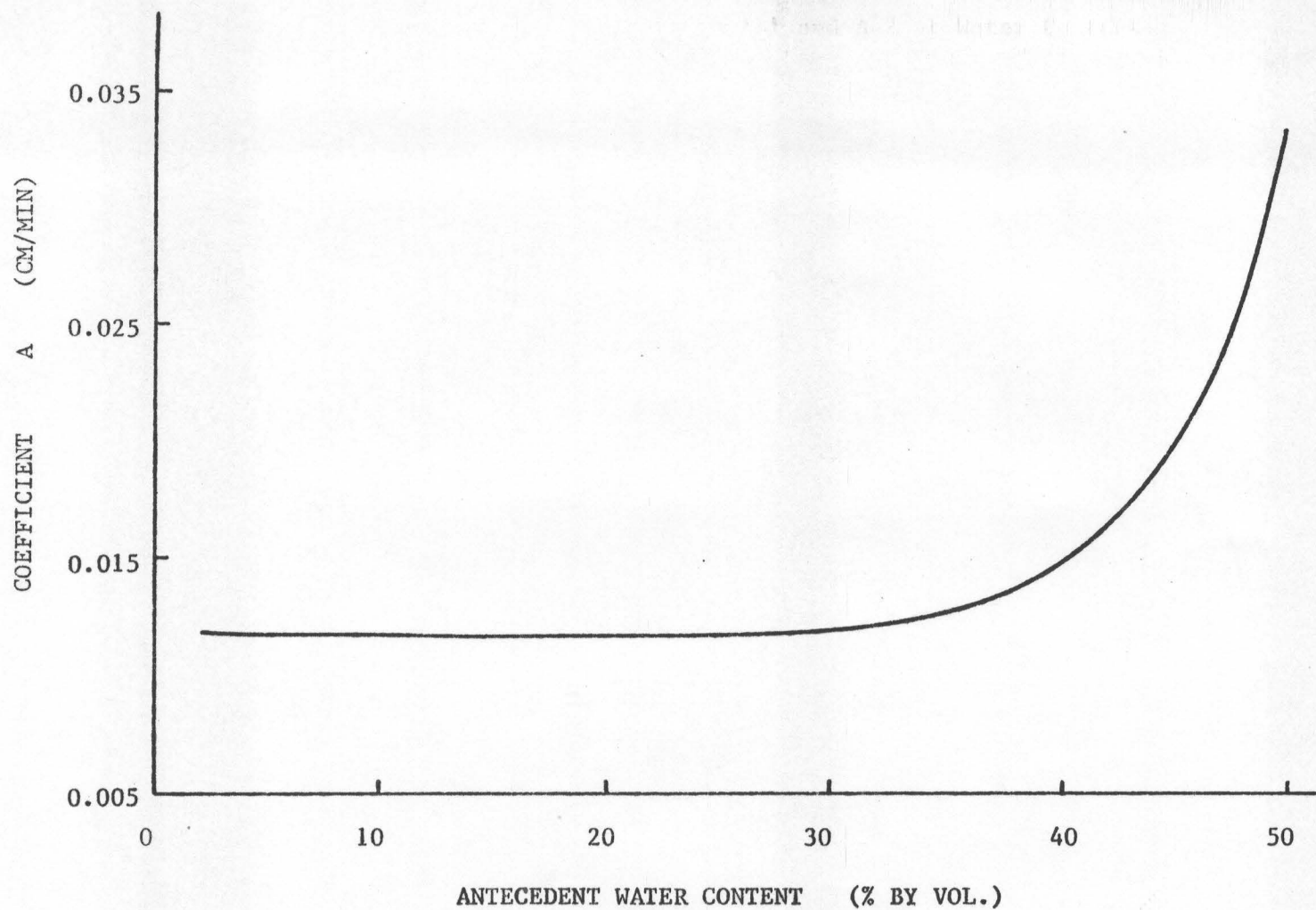


Figure 3.3 Coefficient A of Philip's Infiltration Equation As a Function of Antecedent Soil Water Content. Molokai Soil, HSPA Site A.

Table 3.1 Calculated S and A for Combined Ap1 and Ap2 at Water Contents
Corresponding to Measured S of Ap1 Layer for Seven Field Sites.

Site No.		Soil water content θ % by vol.	Calculated parameters for entire Ap horizon		Measured S for Ap1 S cm/min ^{1/2}	Matching factor for S(θ_o) ---
			S(θ_o) cm/min ^{1/2}	A cm/min ²		
HSPA	A	23	0.348	0.0074	1.29	3.707
	B	23	0.248	0.0034	1.80	7.258
	C	22	0.156	0.0015	1.44	9.231
OP221	E	30	0.384	0.0210	1.21	3.151
	W	30	0.470	0.0149	1.21	2.574
OP410	E	30	0.399	0.0140	1.39	3.484
	W	30	0.416	0.0188	1.42	3.413

different antecedent water contents can be calculated with the Philip equation.

An example of calculated infiltration rate at different antecedent soil water contents is shown in Figure 3.4, and the appropriate parameters, S (matched) and A , used in the calculation are shown in Table 3.2.

Examples of calculated and measured infiltration rates from the same experimental site (HSPA-Site A) at different antecedent soil water contents are shown in Figures 3.5 and 3.6. Two calculated infiltration rate curves are shown. One is the infiltration rate calculated with the unmatched S and the other was calculated with matched S ; the same calculated value of A was used in both cases as previously discussed. The infiltration rate calculated with unmatched S is low compared with the measured and matched results. However, the predictions of infiltration rate were greatly improved when calculated with the matched $S(\theta_0)$. Overall comparisons of calculated and measured cumulative infiltration for all sites are shown in Figure 3.7. The periods of cumulative infiltration used for comparison are from 5 minutes to the approximate time when the wetting front reached the B horizon. The reason for not including the first five minutes for the comparison is the uncertainty in early experimental values; the constant-head in the infiltrometer is difficult to maintain initially. The upper limit of time for which cumulative infiltration is calculated for each comparison is based on the water storage available in the Ap horizon; water-fillable porosity at a given antecedent water content is assumed to be the difference between the water content at "field saturation" and the antecedent

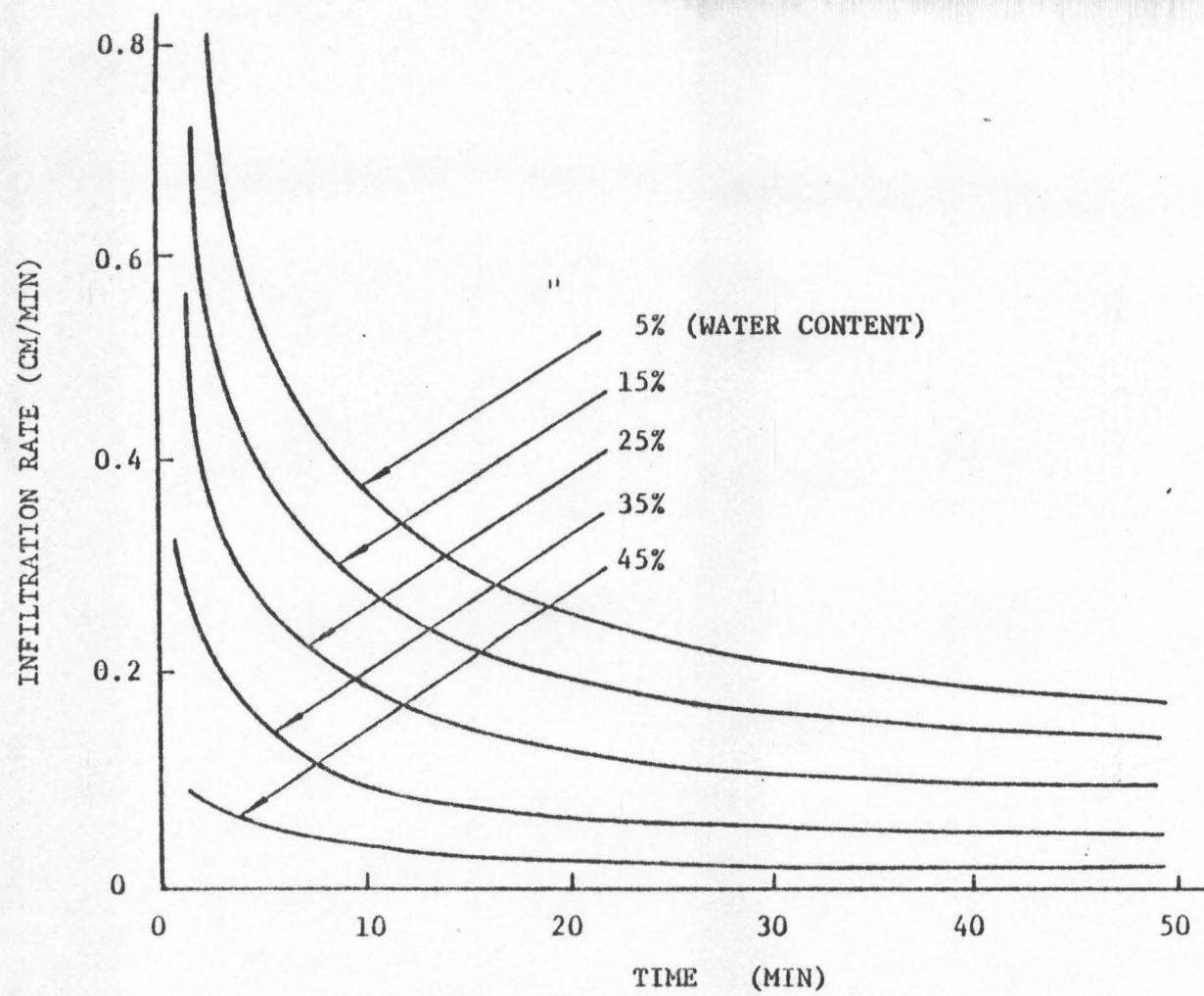


Figure 3.4 Calculated Infiltration Rate At Different Antecedent Soil Water Contents (by vol.). HSPA Site A.

Table 3.2 Parameters Used in Calculating the Infiltration Rate
for HSPA SITE A.

Antecedent Soil Water Content (% by vol.)	Matched Sorptivity (cm/min ^{1/2})	Coefficient A (cm/min ²)
5	2.298	0.012
15	1.739	0.009
25	1.175	0.007
35	0.619	0.005
45	0.115	0.008

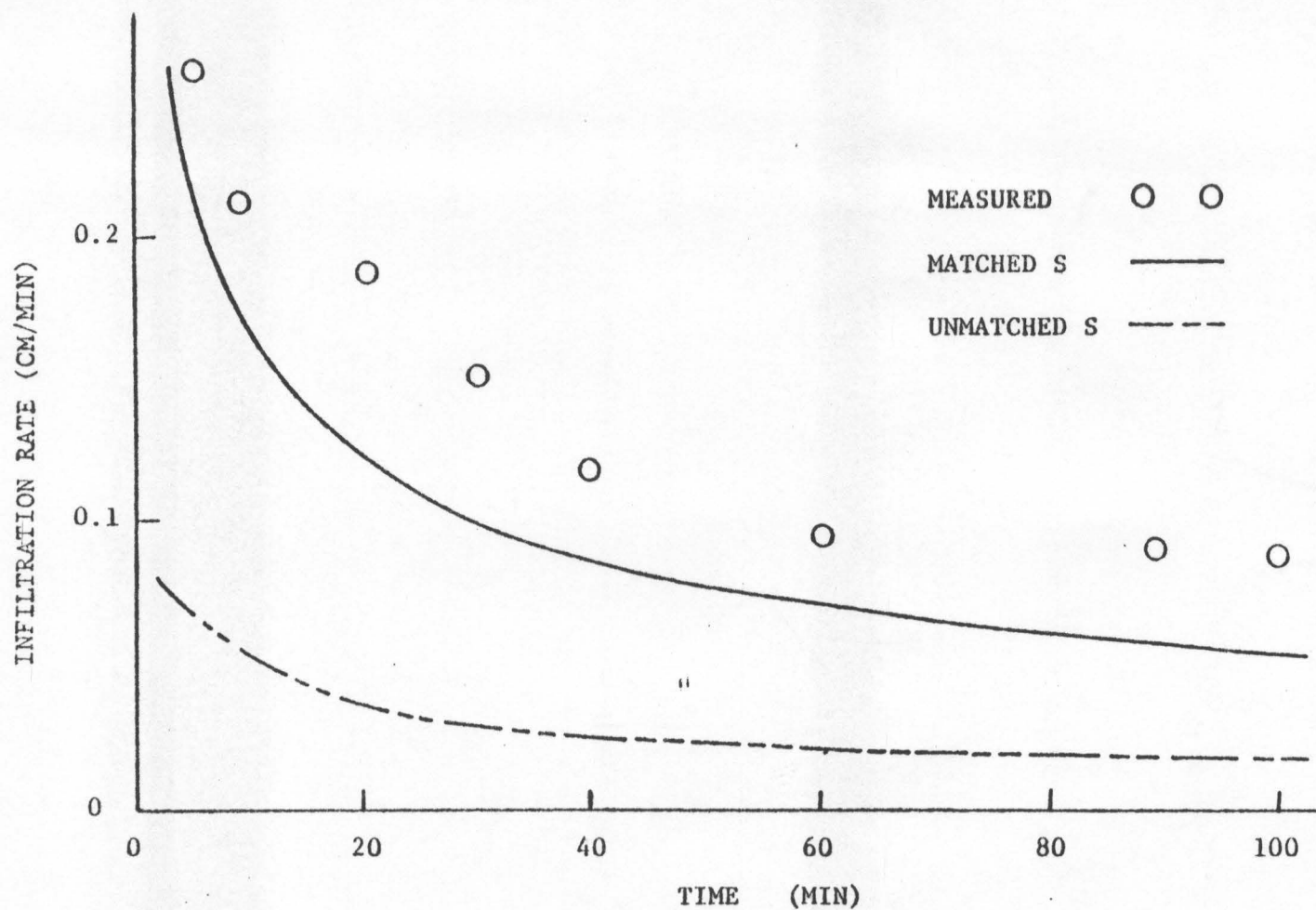


Figure 3.5 Comparison of Measured Infiltration Rate with Rate Curves Calculated by the Philip Equation with Both Matched and Unmatched Sorptivity. Molokai Soil, HSPA Site A, $\theta_0 = 28\%$ (by vol.) (dry run).

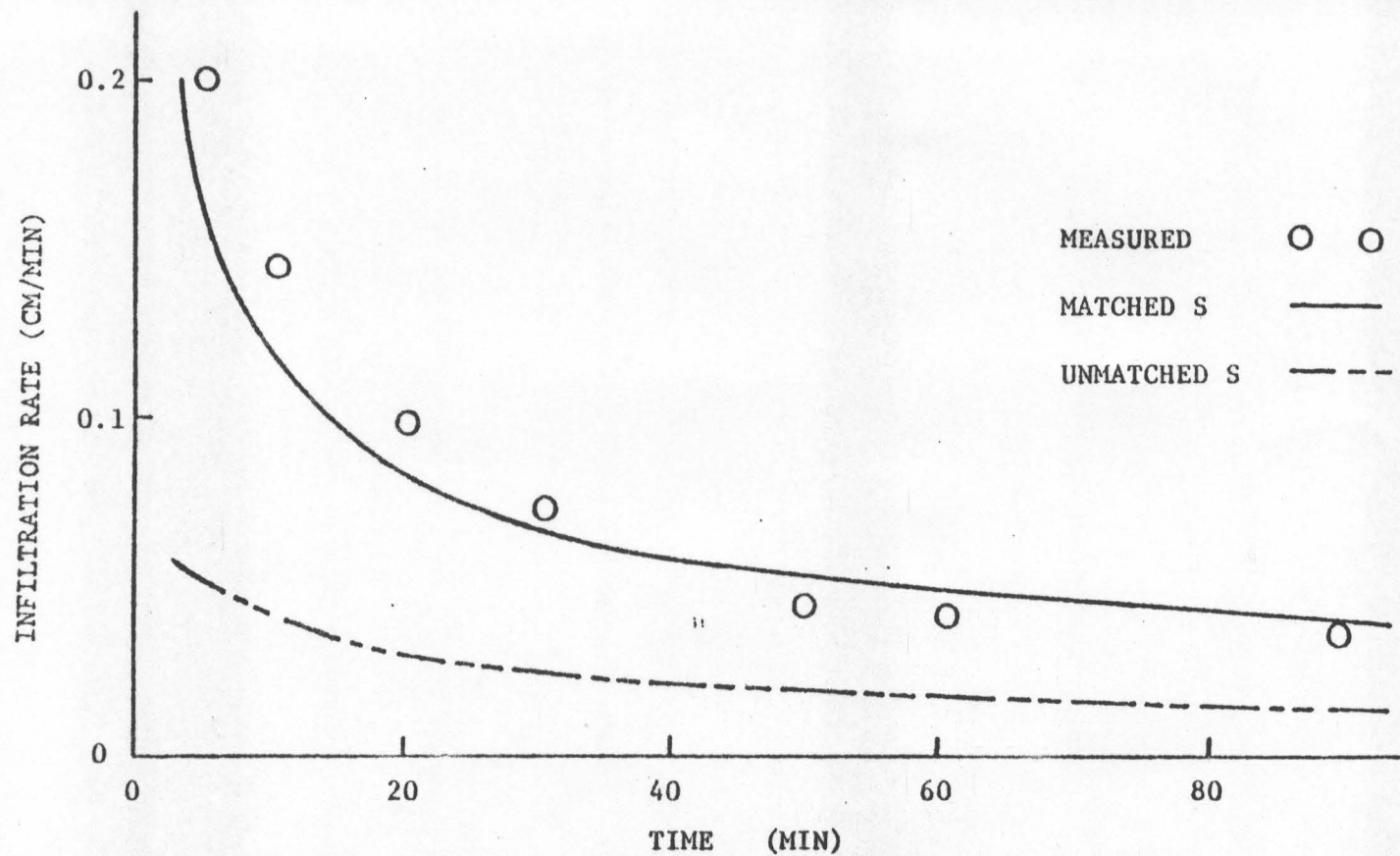


Figure 3.6 Comparison of Measured Infiltration Rate with Rate Curves Calculated by the Philip Equation with Both Matched and Unmatched Sorptivity. Molokai Soil, HSPA Site A, $\theta_0 = 33\%$ (by vol.) (wet run).

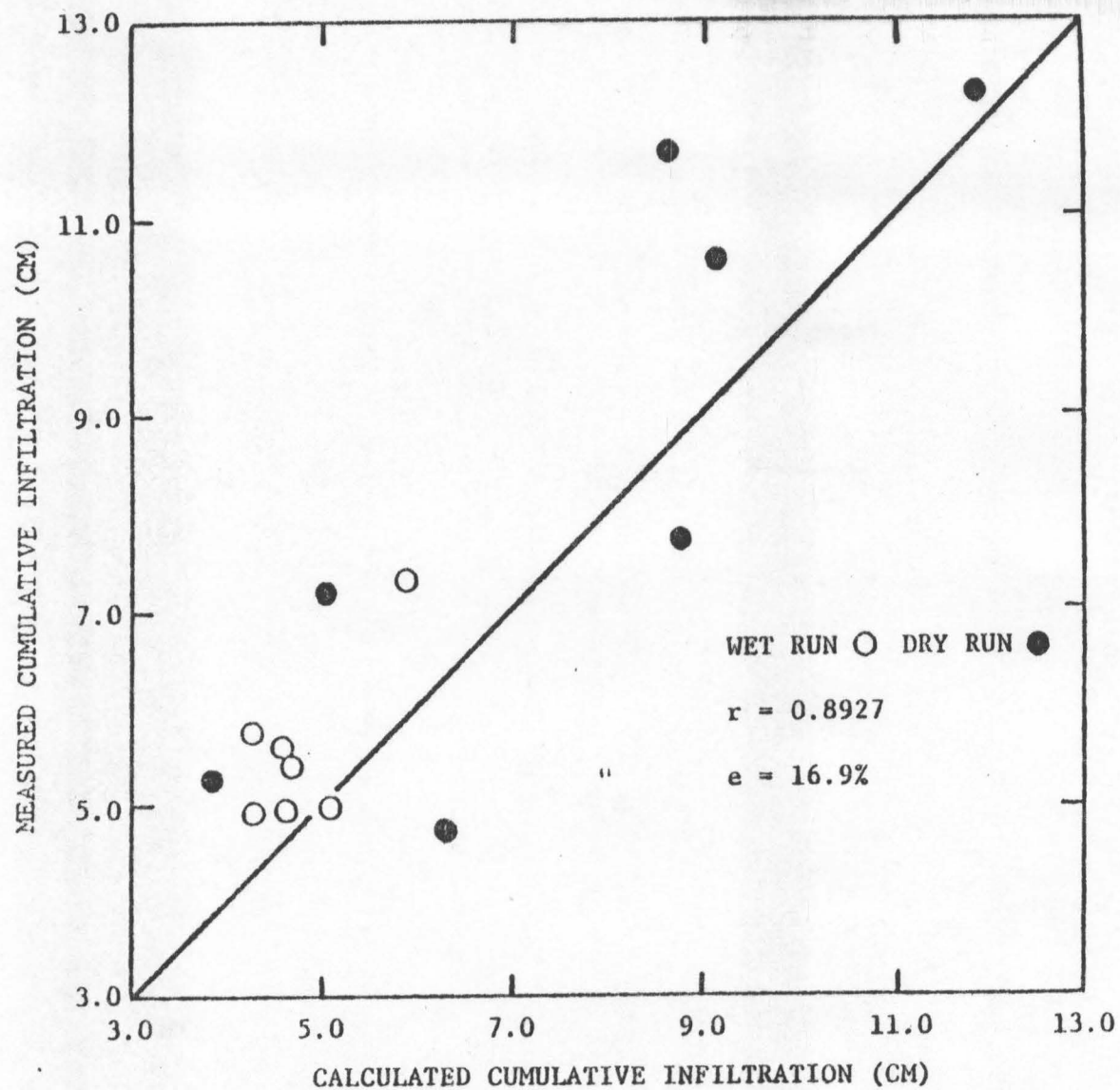


Figure 3.7 Comparison of Calculated (Matched S) and Measured Cumulative Infiltration for Both Dry and Wet Antecedent Conditions at Seven Field Sites.

water content. The correlation coefficient, r , between the calculated and measured cumulative infiltration is 0.89, and the average percentage error [Topping, 1962] is 16.9%.

SUMMARY AND CONCLUSION

In this study, Philip's two-parameter equation is applied for infiltration prediction. The parameters in the equation, S and A , are calculated from matched $K(\theta)$ and $D(\theta)$, for the A_p horizon, obtained from field experiments described in Chapter Two. The A_p horizon is visualized as two layers in order to adequately characterize the hydrologic properties of the surface soil to which the infiltration process is especially sensitive; the sorptivity of the A_{p1} layer was measured independently of the infiltration measurement using Talsma's method. The calculated and measured cumulative infiltration were compared for seven experimental sites. The results showed that the predicted values are reasonably good in comparison with the experimental results, with a correlation coefficient of 0.89 and average percentage of error of 17%.

In this study we conclude that the field-measured sorptivity is a very useful soil physical parameter in characterizing the surface soil. The results confirmed that the surface soil (A_{p1} layer) has an important influence on infiltration. The field measurement of $K(\theta)$ and $D(\theta)$ developed in Chapter Two is comparatively simple, but installation of a tensiometer at each site for measuring water potential may still limit extensive use of the method on a watershed where spatial variability must be assessed. Estimation of $S(\theta_0)$ directly by more simplified

techniques (discussed in Chapter Five) is desirable when only infiltration prediction is required, but prediction of other processes, such as evaporation, may require the $K(\theta)$ and $D(\theta)$ data anyway. If conductivity and diffusivity data are already available for a given location, the method proposed in this study is a good candidate for field use, as it combines results from a rapid, sensitive method of determining sorptivity at the soil surface with corresponding calculations based on fundamental hydrologic properties of the soil profile.

CHAPTER FOUR

PREDICTION OF GREEN-AMPT WETTING FRONT POTENTIAL
AND ASSOCIATED INFILTRATION FROM IN-SITU SOIL
WATER REDISTRIBUTION MEASUREMENTS

INTRODUCTION

Recently, in watershed infiltration analysis, much interest has been directed toward using the Green and Ampt [1911] approach because of its simplicity and encouraging results [e.g. Mein and Larson, 1973; Swartzendruber and Hillel, 1975; Dangler et al., 1976]. Moreover, this simple and empirical approach [Hillel, 1971] can also be extended to consider infiltration into more complicated soils [e.g. Childs, 1967; Childs and Bybordi, 1969; Bouwer, 1969; Hillel and Gardner, 1970; Swartzendruber, 1974; Ahuja, 1974; Youngs and Aggelides, 1976]. The Green-Ampt approach is obtained by applying Darcy's equation to a wetting soil profile with assumptions of vertical flow, and a transmission zone with both uniform water content and uniform hydraulic conductivity with depth. Furthermore, it is assumed that in the wetting soil profile there exists a distinct and precisely definable wetting front; the matric potential at the wetting front is constant regardless of time and position during the infiltration. Therefore, with consideration of the gravity effect, the Green-Ampt approach gives the following simple infiltration equation [Hillel, 1971]:

$$Kt = I - \Delta\theta(H_o - H_f) \ln \left(1 + \frac{I}{\Delta\theta(H_o - H_f)} \right) \quad (4.1)$$

in which,

K is the hydraulic conductivity in the transmission zone, cm/min.

t is time, min.

$\Delta\theta$ is the difference between field saturated and antecedent soil water content, cm^3/cm^3 .

H_o is the pressure head at the water entry surface, cm.

H_f is the matric potential at the wetting front, cm.

I is cumulative infiltration, cm.

Equation (4.1) is an algebraic infiltration equation in which the parameters describe the physical properties of the soil-water system [Philip, 1957]. A major obstacle in using (4.1) is the difficulty of estimating the parameter H_f . Especially in the field, the wetting front potential is difficult to define when the wetting front is diffuse as a result of non-uniform antecedent water content or variation in physical properties with depth.

Bouwer [1964] was the first worker who suggested that H_f can be calculated by

$$H_f = \int_0^{\psi_i} K_r(\psi) d\psi \quad (4.2)$$

where

K_r is the relative hydraulic conductivity, dimensionless

ψ is the matric potential, cm, and

ψ_i is the matric potential at the antecedent water content, cm.

Equation (4.2) has been derived theoretically from Darcy's Law by Morel-Seytoux and Khanji [1974] for two-phase flow, and by Neuman [1976] for water flow only, neglecting air movement.

Several ways of estimating H_f have been proposed. Bouwer [1966] developed an apparatus for in-situ measurement of the air-entry value and used air-entry value to estimate H_f ; Mein and Farrel [1974]

determined wetting front potential by a theoretical justification; Panikar and Nanjappa [1977] redefined H_f by multiplying K_r in (4.2) by relative soil water content; Brakensiek [1977] related H_f to the bubbling pressure which can be obtained from a soil-water characteristic curve; Clapp et al. [1978] derived an empirical equation for estimating H_f . However, most of the methods require a knowledge of either the soil-water characteristic or the hydraulic conductivity-water content relationship. Moreover, most of the required information they used was obtained in the laboratory.

In this study, a simple algebraic equation was derived for calculating H_f based on in-situ soil water redistribution measurements. There is no attempt to compare the calculated H_f with the value obtained by other methods. Instead, the calculated H_f is used to predict infiltration using (4.1), and predicted infiltration is compared with the field-measured results. The method was tested on several field sites on the Wahiawa Plateau, Island of Oahu, Hawaii. The soils of this study are well-aggregated and well-drained Typic Torrox soils.

METHODS

Derivation of Equation for Calculating Wetting Front Potential

In consideration of the low hydraulic conductivity at low soil water contents, instead of using (4.2) for calculating H_f , Mein and Larson [1973] proposed an alternative, Equation (4.3).

$$H_f = \frac{\int_{K(\theta_i)}^{K(\theta_s)} \psi dK(\theta)}{K(\theta_s) - K(\theta_i)} \quad (4.3)$$

where,

K is hydraulic conductivity, cm/min.
 θ_s is saturated soil water content, cm^3/cm^3 .
 θ_i is antecedent soil water content, cm^3/cm^3 .

The expression of (4.2) and (4.3) are essentially the same when K is considerably small.

Furthermore, we have assumed that during the soil-water redistribution period both soil water content and soil water potential in the profile can be expressed as functions of time as shown in Chapter Two, such that,

$$\theta = at^b \quad (4.4)$$

$$\psi = mt^n \quad (4.5)$$

in which,

θ is soil water content, cm^3/cm^3 .
 ψ is soil water potential, cm.
 t is redistribution elapsed time, min.
 a , b , m , and n are constants.

Based on the assumption of unit gradient, the rate of change of soil water content in the profile during redistribution can be used to calculate hydraulic conductivity as shown in Chapter Two, such that

$$K_L = -L \frac{d\theta}{dt} \quad (4.6)$$

where,

L is depth of soil profile, cm.
 K_L is hydraulic conductivity at depth L , cm/min.

Substituting (4.4) and (4.5) into (4.6), the following equation can be obtained.

$$K = C_1 \psi^{C_2} \quad (4.7)$$

where

$$C_1 = -L abm^{-\left(\frac{b-1}{n}\right)}$$

$$C_2 = \left(\frac{b-1}{n}\right)$$

Again, substituting (4.7) into (4.3), we have

$$H_f = \frac{1}{K_s - K_i} \int_{K_i}^{K_s} \left(\frac{K}{C_1} \right)^{\frac{1}{C_2}} dK \quad (4.8)$$

Integrating (4.8) with the assumption that $K = K_s$ when $\psi = \psi_s$ (s denotes a saturated condition), and also using the relation of (4.4) and (4.6), H_f can be expressed as a function of water content; that is,

$$H_f = m \left(\frac{\theta_s}{a} \right)^{\frac{n}{b}} \frac{b-1}{b+n-1} \left(\frac{1 - \left(\frac{\theta_i}{\theta_s} \right)^{\frac{b+n-1}{b}}}{1 - \left(\frac{\theta_i}{\theta_s} \right)^{\frac{b-1}{b}}} \right) \quad (4.9)$$

Equation (4.9) is the working equation for calculating the wetting front matric potential in this study. The specific advantages of (4.9) are: (a) it is an algebraic equation in which θ is the only independent variable, and (b) H_f can be obtained at any antecedent water content, if the parameters a , b , m , n , and θ_s can be determined. The detailed derivation of (4.9) is available in Appendix III.

Experimental Procedures

In order to determine the parameters in (4.9), field experiments were conducted on the Wahiawa Plateau of the Island of Oahu. All of the experimental sites were in agricultural fields. Infiltration measurements were conducted with double-ring infiltrometers with a 2-cm head maintained by controlling the water inlet. Water application was continued approximately one hour beyond the time that an apparent steady infiltration rate was observed. The infiltration measurement

was conducted on dry soil first, and subsequently on wet soil about four days later.

The starting time of redistribution was defined as the time when applied water was just disappearing from the soil surface. The soil water content data during the redistribution period were obtained gravimetrically from soil samples obtained between the inner and outer rings. Volumetric water contents were calculated using bulk density and particle density data obtained from soil cores taken from the experimental site after redistribution measurements were terminated. The water potentials during redistribution were obtained from tensiometer readings; the tensiometer was installed at the center of the inner infiltrometer ring. Details of site preparation, infiltration and redistribution measurements are given in Chapter Two. The entire experiment was conducted within about 10 days.

Determination of the Parameters in Equation (4.9)

Parameters for water redistribution equations (4.4) and (4.5) were determined by regression with experimental data for soil water content versus redistribution elapsed time and soil water potential versus redistribution elapsed time. In this study, 85% of total porosity is used as the "field saturated" soil water content, θ_s . The details of determination of the parameters in (4.9) and the reason for using 85% of total porosity as "field saturated" soil water content are also given in Chapter Two.

Calculation of Infiltration Using Equation (4.1)

Equation (4.1) cannot be solved explicitly for I due to the I in the logarithm term. Therefore, (4.1) has to be solved either by a trial and error method using a computer [Hornbeck, 1975], or by a graphical method. The graphical method used in this study is as follows: Separate (4.1) into the left-hand-side (LHS) and the right-hand-side (RHS). On the LHS, Kt is independent of I . This means that for a given t , when we plot Kt versus I on linear scale paper, Kt is a constant regardless of what I will be. Then we can assume a series of I values and calculate the corresponding values of the RHS of (4.1). If we plot the values of RHS versus I on the same graph as Kt versus I , the projection of the intersection of the two curves on the I axis provides the solution of I in (4.1). An example of solving I is shown in Appendix V.

In the derivation of (4.1), it is assumed that the soil profile is uniform. In fact, the field profile varies with depth and is thus divided into A_p and B horizons as shown in Chapter Three. In the calculation of infiltration, in order to satisfy (4.1), we can deal only with the relatively uniform A_p horizon; this means that (4.1) is valid only for values of t less than the time when the wetting front reaches the interface of the A_p and B horizons.

The total amount of water infiltrated in the A_p horizon can be calculated as

$$I_m = \Delta\theta L_{A_p} \quad (4.10)$$

where,

I_m is the measured amount of water infiltrated in the A_p horizon, cm.

L_{Ap} is the depth of Ap horizon, cm.
 $\Delta\theta^p$ is the water-fillable porosity, which for our purpose is assumed to be the difference between 85% of total porosity and the antecedent soil water content, cm^3/cm^3 .

Since I_m can be determined, the approximate time, t , that is required by the wetting front in the profile to reach the depth L_{Ap} can be obtained from the experimental cumulative infiltration results.

RESULTS AND DISCUSSION

Calculation of Wetting Front Potential

The constants from the regression of (4.4) and (4.5) on experimental data for each experimental site are tabulated in Table 4.1. The details of determination of these constants and examples of experimental and regressed results are shown in Chapter Two. For all experimental sites, the correlation coefficient, r , between regressed and measured results for θ versus t exceeded -0.95 and for ψ versus t exceeded -0.98.

Examples of calculated K (from Chapter Two), and calculated H_f from (4.9) versus θ and ψ are shown in Figure 4.1. The hydraulic conductivity is very small and approaches zero at a water content of 35% by volume. On the other hand, the calculated H_f , from (4.9), increases sharply with even slight desaturation, and reaches a maximum at a water content of about 35% by volume.

In Equation (4.9), the parameter n is always positive, but less than 1.0, due to the increasing negative value of water potential versus time during drainage, i.e. ψ becomes more negative with time in (4.5). Also, b (generally less than -0.1, see Table 4.1) is always negative because of decreasing water content in the soil profile. Therefore,

Table 4.1 Parameters for Water Redistribution Equations (4.4) and (4.5)
Determined by Regression with Experimental Data from Seven
Sites, and the Calculated Wetting Front Potential
from Equation (4.11)

Site		a	b	m	n	θ_s	H_f (cm)
HSPA	A	0.6079	-0.0595	-8.5570	0.3259	0.504	-34.50
	B	0.6602	-0.0633	-1.1095	0.5505	0.520	-18.33
	C	0.6071	-0.0611	-4.8220	0.3807	0.482	-31.67
OP221	E	0.5895	-0.0601	-6.6110	0.3555	0.530	-18.66
	W	0.7132	-0.0769	-8.1103	0.3446	0.522	-48.29
OP410	E	0.7058	-0.0797	-5.8426	0.3718	0.532	-33.02
	W	0.6110	-0.0601	-12.1452	0.2761	0.536	-29.97

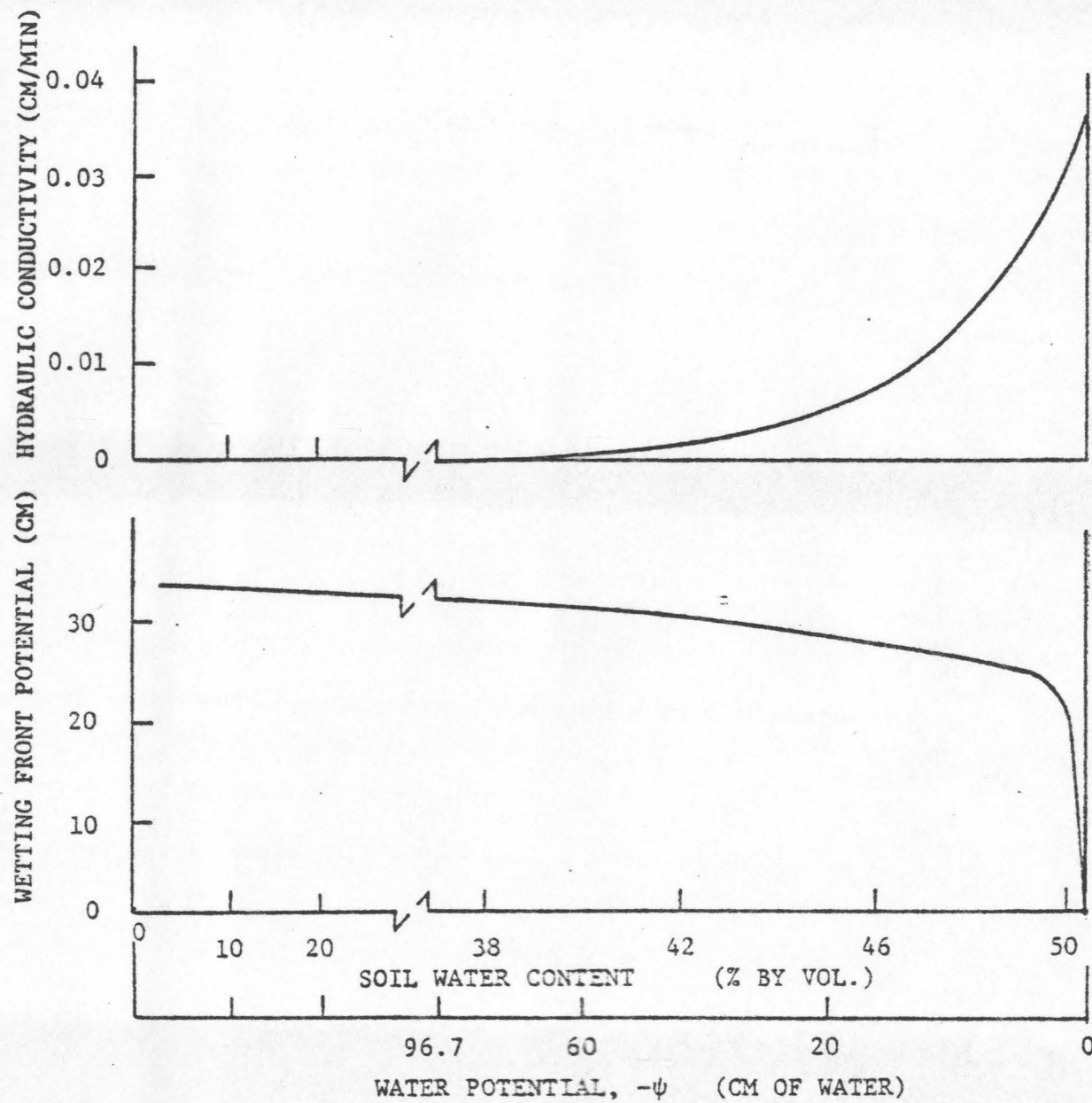


Figure 4.1 The Relationships of Hydraulic Conductivity and Wetting Front Potential; HSPA Site A.

the power terms for (θ_i/θ_s) in (4.9) are always positive and generally larger than 5.0. Since (θ_i/θ_s) is always much less than 1.0 for small θ_i , the term (θ_i/θ_s) in (4.9) can be eliminated with a little over for low antecedent water contents. This implies that H_f is essentially constant for antecedent water contents below field capacity. The soil water content at 48 hours after initiation of redistribution is between 36 to 39% (by volume) for all of the experimental soils in this study. Mathematically, therefore, (4.9) explains why H_f is virtually constant over a wide range of antecedent soil water contents [Mein and Larson, 1973; and Mein and Farrel, 1974]. For water contents lower than field capacity, the term (θ_i/θ_s) in (4.9) can be neglected, and finally (4.9) becomes

$$H_f = m \left(\frac{\theta_s}{a} \right)^{\frac{n}{b}} \left(\frac{b-1}{b+n-1} \right) \quad (4.11)$$

The calculated H_f values using (4.11) for all experimental sites are shown in the last column of Table 4.1.

Predicted Infiltration

The Green-Ampt approach is more satisfactory for calculation of infiltration in dry soil than in wet soil [Hillel and Gardner, 1970]. This may be because the wetting front moving down through wet soil is more diffuse than in dry soil; the movement of the wetting front essentially does not conform to the Green-Ampt piston-type flow assumption when the soil is too wet.

In this study we are interested in calculating wetting front potentials for various soils and the associated cumulative infiltration with time. Only the dry antecedent condition of each soil will be used

in the evaluation of this prediction equation, in that the Green-Ampt equation is less likely to be valid for wet soils. Green-Ampt predictions on initially dry soil will also be compared with calculations obtained by Philip's equation (in Chapter Three).

The parameters required in (4.1) for calculating infiltration for all seven experimental sites are shown in Table 4.2. K_s is the "field saturated" hydraulic conductivity (field measured "steady" flux divided by the appropriate measured gradient), which is required by the Green-Ampt equation [Bouwer, 1978]. The fillable porosity, $\Delta\theta$, is the difference between antecedent soil water content and 85% of total porosity, which is assumed to be the field "saturated" water content in our study (see Chapter Two). The antecedent soil water content was obtained gravimetrically in the field just before infiltration measurements.

Both the measured cumulative infiltration, I_m , and calculated Green-Ampt cumulative infiltration, I_c , in Table 4.2 correspond to the infiltration period from 5 minutes elapsed time to the time, t , designated in Table 4.2. The first 5 minutes of infiltration is neglected for the purpose of these comparisons because of uncertainties in the measured inflow rates soon after initiation of infiltration. The calculated upper limit of time, t , for each site is based on the estimated water storage available in the Ap horizon.

I_m and I_c for dry antecedent conditions (D) in Table 4.2 are plotted in Figure 4.2 in relation to a 1:1 line for these values to indicate the accuracy of the Green-Ampt prediction of infiltration for the seven sites. The average deviation of predicted cumulative

Table 4.2 Parameters for Equation (4.1) and the Comparisons of Calculated and Measured Results for All Seven Sites.

Site	K_s cm/min	L_{Ap} cm	t min	$\Delta \theta$ cm ³ /cm ³	H_f cm	I_m cm	I_c cm
HSPA A (D)	0.0411	40	49	0.224	-34.50	7.10	5.2
(W)	0.0411	40	68	0.174	-34.50	4.40	6.2
HSPA B (D)	0.0217	40	99	0.160	-18.33	4.70	4.4
(W)	0.0217	40	60*	0.170	-18.33	5.40	5.2
HSPA C (D)	0.0083	40	60*	0.152	-31.67	5.23	3.3
(W)	0.0083	40	60*	0.152	-31.67	4.84	3.5
OP221 E (D)	0.1451	50	45	0.290	-18.66	11.67	10.6
(W)	0.1451	50	55	0.190	-18.66	7.27	11.0
OP221 W (D)	0.1010	50	70	0.292	-48.29	12.30	15.3
(W)	0.1010	50	60	0.142	-48.29	5.70	10.7
OP410 E (D)	0.0933	30	40	0.313	-33.02	7.67	8.0
(W)	0.0933	30	45	0.183	-33.02	4.95	7.8
OP410 W (D)	0.1251	40	40	0.306	-29.97	10.57	9.8
(W)	0.1251	40	45	0.166	-29.97	5.57	8.9

W and D in parentheses denote wet and dry infiltration runs.

* assumed.

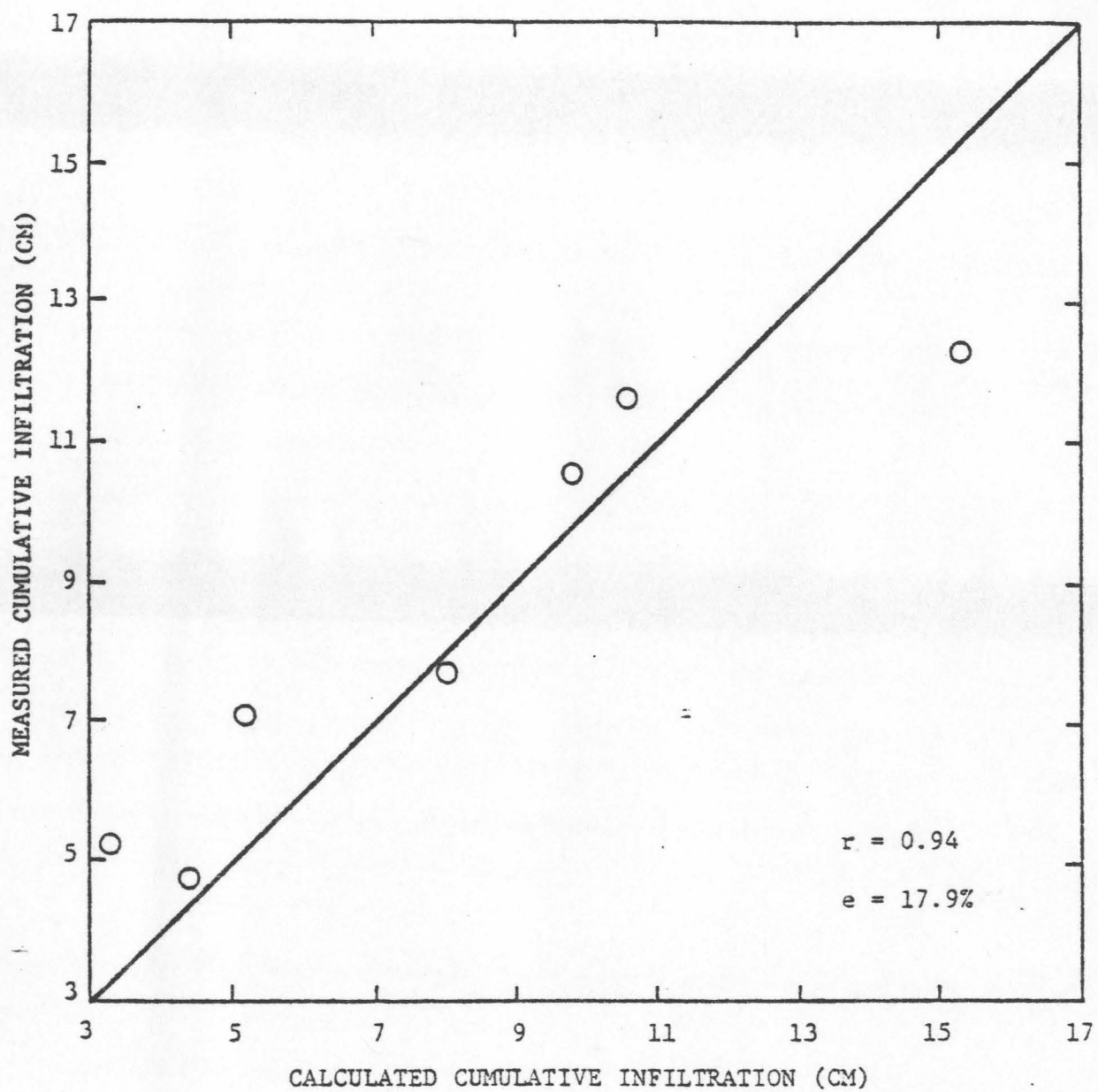


Figure 4.2 Comparison of Measured and Calculated (by Green-Ampt Approach) Infiltration for Infiltration in Dry Soils.

infiltration from that measured at each site is expressed as an average percentage error, e [Topping, 1966],

$$e = \left[\frac{1}{n} \sum \frac{|I_m - I_c|}{I_m} \right] \cdot 100 \quad (4.12)$$

in which

n is the number of sites.

For the data in Figure 4.2 $e = 17.9\%$, i.e. the predicted values differ from the measured values by an average of 17.9%. The corresponding correlation coefficient for I_m and I_c is $r = 0.94$. These results are promising in view of a likely inconsistency between the Green-Ampt assumption of a uniform soil profile and the actual variation of physical properties with depth in those field soils (see Chapter Three). As expected the predicted infiltration for wet antecedent conditions was less accurate, $e = 47\%$ and $r = 0.71$, confirming that the Green-Ampt approach is not appropriate for wet soils.

It is of interest to compare the relative accuracy of predictions by the Green-Ampt equation and the Philip 2-term equation (in Chapter Three). Predicted and measured cumulative infiltration curves for HSPA Site A are compared in Figure 4.3. For this site the Philip equation appears to give the best results, and an analysis of all predictions by the Philip equation confirms its overall superiority for the locations in this study. The Philip-equation predictions for the sites listed in Table 4.2 had an average error of 21% for dry conditions and only 13% for wet conditions. Thus the dry soil predictions by the Philip equation have only about a 3% higher error than the Green-Ampt calculations, but Philip-equation predictions on wet soil had a relatively low error, 13% versus 47% for the Green-Ampt calculation.

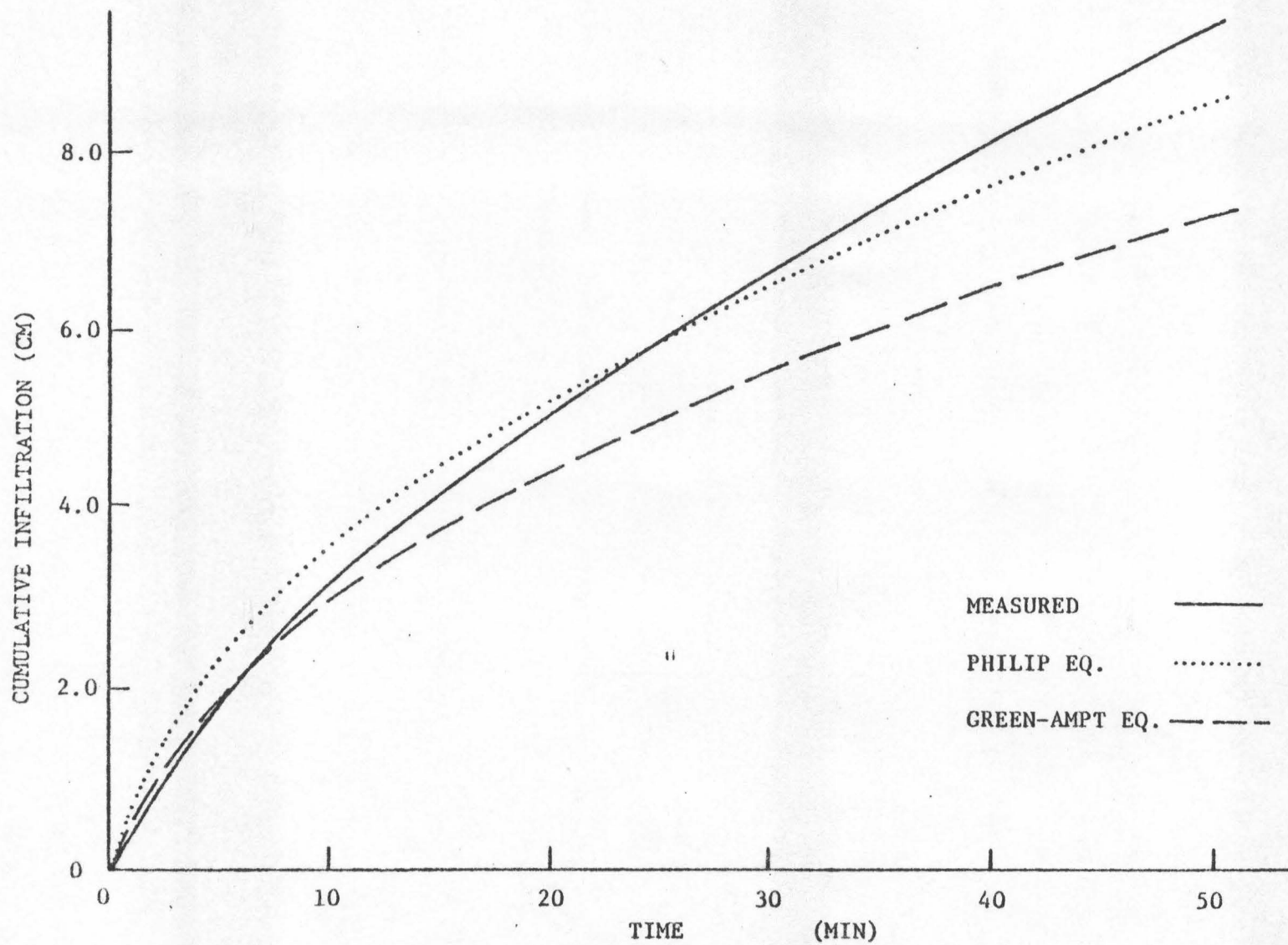


Figure 4.3 Comparison of Calculated and Measured Cumulative Infiltration for Molokai Soil, HSPA Site A.

CONCLUSIONS

Several conclusions can be drawn in this study:

1. A simple algebraic equation, (4.9), is derived for estimating wetting front matric potential as a function of antecedent water content, based on in-situ soil water redistribution measurements.
2. The wetting front potential at low antecedent water content (e.g. water content lower than field capacity) is essentially a constant, which can be explained mathematically by the derived equation.
3. The Green-Ampt approach was satisfactory for infiltration prediction on dry soil but not on wet soil. The correlation coefficient between calculated and measured cumulative infiltration for dry soil is $r = 0.94$ with an average percentage of error of 17.9%. For wet soil, $r = 0.71$, but with an average percentage of error over 47%.
4. The 2-term Philip equation is superior to the Green-Ampt equation in watershed infiltration analysis because Philip's equation predicted infiltration equally well on wet and dry soil. An abrupt wetting front is required for Green-Ampt theory but not for the Philip equation. Prediction error on dry soil was comparable for both methods.
5. In this study, there is no attempt to compare the calculated H_f with the values obtained by other methods. But, the Green-Ampt infiltration prediction results indicate that values of H_f calculated by (4.9) are reasonable.

CHAPTER FIVE
APPLICATION OF FIELD-MEASURED SORPTIVITY FOR
SIMPLIFIED INFILTRATION PREDICTION

INTRODUCTION

In watershed simulation a major hindrance in predicting runoff from a watershed is the uncertainty in characterizing infiltration. The difficulty of predicting infiltration is due mainly to the variation of infiltration-related soil physical properties from site to site in the field. This implies that in order to predict infiltration in a watershed, it is first necessary to characterize pertinent soil properties for the entire watershed. The problems of characterizing these watershed soil properties have been discussed elsewhere (e.g. Fleming and Smiles, 1975; see Chapter One). However, as concluded by Klute [1973], the major problems in applying Darcy-based flow theory are the measurements of hydraulic conductivity, $K(\theta)$, and diffusivity, $D(\theta)$, and also how to define the initial and boundary conditions of the flow situation. If the infiltration-related soil physical properties, $K(\theta)$ and $D(\theta)$, and the initial and boundary conditions of the flow situation can be determined, the theoretical flow equation can be solved either by numerical or analytical methods. However, the mathematical computations involved in solving the theoretical equation are fairly complicated. Therefore, in order to simplify the infiltration problem, researchers have introduced a number of simple algebraic

infiltration equations, for example the Green-Ampt equation [1911], the Kostiaikov equation [1932], the Horton equation [1940], the Philip equation [1957], the Holtan equation [1961], the Talsma-Parlange equation [1972] and the Collis-George equation [1977].

These simple algebraic equations are physically based or empirical. In order to apply these equations, the equation parameters must first be determined. Some of the physically based parameters can be measured in the field, e.g. the saturated hydraulic conductivity, K_s , in the Green-Ampt equation, or the sorptivity, S , in the Philip and the Talsma-Parlange equations. But most of the parameters for empirical equations are determined from regression of the infiltration equation on the experimental data. In general, the regressed parameters are good only for the particular set of data from which they originated and cannot be used with confidence for other cases. This implies that most of the simple algebraic equations are not adequate for infiltration prediction on a range of soils for which both temporal and spatial variability will be encountered.

In short, in watershed infiltration analysis, we need a reasonably accurate prediction equation which can accommodate spatial variability. Also, the method of determining equation parameters should be simple and consistent, and the parameters should be sufficiently sensitive to represent significant variations in infiltration associated with soil differences in a watershed.

Much attention has been given to the Philip 2-term equation. In Philip's equation, there are two parameters, the sorptivity, S , and the coefficient A . Sorptivity is the most important simple quantity

governing the early portion of infiltration [Philip, 1957]. It varies for different soils, depends on the structure and the pore-size distribution of the soil, and is also influenced by antecedent water content [Bouwer, 1978]. A problem in using Philip's equation is the uncertainty in estimation of the parameter A [Youngs, 1968; Swartzendruber and Youngs, 1972; and Parlange, 1975]. Recently, a 3-term equation, which is similar to Philip's equation, has been developed by Talsma and Parlange [1972]. Even though the Talsma-Parlange equation contains three terms, it requires only two parameters which characterize the soil, the sorptivity, S , and the saturated hydraulic conductivity, K_s . Both S and K_s can be measured in the field.

In this study, the Talsma-Parlange equation will be used to predict infiltration. The parameters S and K_s in the equation were obtained directly in the field. The nature of the statistical distribution of the measured sorptivities is tested by the Kolmogorov-Smirnov method. Because sorptivity varies with antecedent water content, θ_0 , a linear approximation of the S - θ_0 relation is assumed for infiltration prediction. The method was tested on the Molokai and Lahaina soil series at seven soil locations with 26 infiltration measurements. All experimental sites are located in cultivated soils on the island of Oahu, Hawaii.

DESCRIPTION OF TALSMA-PARLANGE INFILTRATION EQUATION

Parlange [1971] developed Equation (5.1), using an integral method, to express the relationship between infiltration rate, i , and time, t .

$$t = \int_{\theta_0}^{\theta_s} (\theta - \theta_0) DK^{-2} \left[\ln \left(\frac{i-K}{i} \right) + \frac{K}{(i-K)} \right] d\theta \quad (5.1)$$

where

θ_s is saturated soil water content, cm^3/cm^3 .

θ_0 is antecedent soil water content, cm^3/cm^3 .

D is diffusivity, cm^2/min .

K is hydraulic conductivity, cm/min .

According to Talsma and Parlange [1972], in order to simplify (5.1), it is assumed that both D and $\frac{dK}{d\theta}$ vary in the same way with θ , and are almost proportional. Therefore, $(\theta - \theta_0)D$ in (5.1) can be approximated by

$$(\theta - \theta_0)D \approx \frac{dK}{d\theta} \left(\frac{\int_{\theta_0}^{\theta_s} (\theta - \theta_0) D d\theta}{\int_{\theta_0}^{\theta_s} \left(\frac{dK}{d\theta} \right) d\theta} \right) \quad (5.2)$$

In such a case, the following equation can be obtained from (5.1):

$$2\tau = \ln \left(\frac{1+\lambda}{\lambda} \right) - \left(\frac{1}{1+\lambda} \right) \quad (5.3)$$

where,

$$\tau = \frac{(K_s - K_i)^2 t}{S^2} \quad (5.4)$$

and

$$\lambda = \frac{i - K_s}{K_s - K_i} \quad (5.5)$$

where K_i and K_s are hydraulic conductivity at antecedent and saturation water content, respectively.

Practically, K_i is negligibly small compared to K_s [Talsma and Parlange, 1972]. Therefore (5.4) and (5.5) become,

$$\tau = \frac{K_s^2 t}{S^2} \quad (5.6)$$

$$\lambda = \frac{i - K_s}{K_s} \quad (5.7)$$

Substituting (5.6) and (5.7) into (5.3), the following equation can be obtained,

$$\frac{2 K_s^2 t}{S^2} = \ln \left(\frac{i}{i - K_s} \right) - \frac{K_s}{i} \quad (5.8)$$

(5.8) is essentially equivalent to Equation (2) in Parlange [1975], when H_0 in that equation is assumed to be zero.

For a uniform soil profile, the infiltration rate is essentially a function of the velocity of wetting-front advance, such that

$$i = \Delta\theta \frac{dL}{dt} \quad (5.9)$$

where $\Delta\theta$ is fillable porosity, which is equal to the difference between "field saturation" and antecedent water content in the context of our study.

Substituting (5.9) into (5.8) and integrating (5.8), (5.10) can be obtained. (Equation (5.10) is equal to Equation (5) in Parlange [1975], when $H_0 = 0$).

$$\Delta\theta L = \frac{S^2}{2K_s} \ln \frac{i}{i - K_s} \quad (5.10)$$

Comparing Parlange's Equation (2) (in Parlange [1975]) with the Green-Ampt equation (also see Philip, 1957; Youngs, 1968; and Swartzendruber and Youngs, 1972), for small times, the sorptivity, S , and wetting front potential, H_f , can be related as

$$S^2 = -2\Delta\theta K_s H_f \quad (5.11)$$

Again, substituting (5.11) in (5.10), (5.10) becomes

$$-\frac{L}{H_f} = \ln \frac{i}{i - K_s} \quad (5.12)$$

or,

$$\exp \left(\frac{L}{H_f} \right) = 1 - \frac{K_s}{i} \quad (5.13)$$

Substituting (5.12) and (5.13) into (5.8), we finally have

$$\frac{2 K_s^2 t}{S^2} = -\frac{L}{H_f} + \exp \left(\frac{L}{H_f} \right) - 1$$

or,

$$\frac{2 K_s^2 t}{S^2} = \frac{2 \Delta \theta K_s L}{S^2} + \exp \left(-\frac{2 \Delta \theta K_s L}{S^2} \right) - 1 \quad (5.14)$$

Equation (5.14) is exactly the same as Equation (3) of Parlange [1977], when $I = \Delta \theta L$ (where I is the cumulative infiltration).

The time expansion of Parlange's Equation (3), gives the exact solution of Parlange's Equation (7) [Parlange, 1977], that is

$$\frac{2 K_s I}{S^2} \left(\frac{2 K_s^2 t}{S^2} \right)^{-\frac{1}{2}} = \sqrt{2} + \frac{1}{3} \left(\frac{2 K_s^2 t}{S^2} \right)^{\frac{1}{2}} + \frac{\sqrt{2}}{18} \left(\frac{2 K_s^2 t}{S^2} \right) \quad (5.15)$$

Equation (5.15) (essentially Equation (7) in Parlange [1977]) can be simplified to solve explicitly for cumulative infiltration, I , as a function of time.

$$I = S t^{\frac{1}{2}} + \frac{1}{3} K_s t + \frac{1}{9} \frac{K_s^2}{S} t^{\frac{3}{2}} \quad (5.16)$$

The corresponding infiltration rate, i , is given by

$$i = \frac{1}{2} S t^{-\frac{1}{2}} + \frac{1}{3} K_s + \frac{1}{6} \frac{K_s^2}{S} t^{\frac{1}{2}} \quad (5.17)$$

Equation (5.16) is the working equation for calculating cumulative infiltration in this study. The similarity between (5.16) and the time expansion solution by Philip [1957] is notable.

Equations (5.16) and (5.17) are promising equations for practical infiltration prediction. They are physically based, having physically meaningful parameters, S and K_s , which can be independently measured in the field with relative ease. The equations appear to predict infiltration with sufficient accuracy for moderate times in most practical cases [Talsma and Parlange, 1972]. Thus, (5.16) has two principal advantages over the Philip 2-term equation used in Chapter Two: (a) it avoids the need for $K(\theta)$ and $D(\theta)$, which are required to calculate the parameter A in Philip's equation, and (b) it is accurate for longer times than the Philip equation.

PROCEDURES

Determination of Sorptivity

Indirect calculation method: There are a number of indirect calculation methods for determining sorptivity, for example Philip's method [1955]. A very valuable review of literature concerning calculation of sorptivity has been presented by Brutsaert [1976]. Most of the methods require a knowledge of infiltration-related soil physical properties, $K(\theta)$ and $D(\theta)$, and also require a fairly complicated mathematical computation. The measurements of $K(\theta)$ and $D(\theta)$ are laborious and expensive. Thus, for watershed infiltration analysis it is almost impossible to characterize these physical properties of the entire watershed. Accordingly, the method of calculating sorptivity from measured $K(\theta)$ and $D(\theta)$ can be applied only at a few selected sites, but it is not recommended as a routine method of obtaining sorptivity [Dirksen, 1977].

Direct field measurement: A very simple field method of measuring sorptivity was developed by Talsma [1969]. Talsma's method is based on the assumption that at the very early portion of infiltration, the second term of the right-hand side of Philip's two-parameter equation can be neglected. Therefore, if one plots the early portion of experimental cumulative infiltration versus the square root of the elapsed time on normal scale paper, the sorptivity of the appropriate antecedent soil conditions can be obtained from the slope of the curve. Since this method is simple and rapid, many measurements can be made with limited funds and labor for watershed characterization.

Proposed simplified method--a linear approach: Talsma's field measurement can only provide one-point information per measurement. In Figure 3.2 of Chapter Three, the value of S at saturation is zero. Moreover, the matched sorptivity-water content relation was almost linear. In order to obtain S as a continuous function of θ , we can approximate the S - θ relation by a linear function. $S(\theta_0)$ is obtained, therefore, by passing a straight line from $S = 0$ at saturation through the S value measured in the field by the Talsma method at the existing antecedent water content.

Statistical Analysis

In a large watershed, in order to determine a representative value for a given hydrologic parameter, it is necessary to first determine the form of the distribution of the parameter. For example, if the parameter observations are normally distributed, an arithmetic mean is used. On the other hand, a geometric mean is suggested if the parameter observations are from a log-normal population.

There are a number of equations which can be used for calculating the empirical cumulative distribution function [Chow, 1968; and Haan, 1977]. In this study, the Kolmogorov-Smirnov test was employed for a goodness-of-fit test, and the California method was used to calculate the simple cumulative distribution [Benjamin and Cornell, 1970].

The detailed procedures for the Kolmogorov-Smirnov test are given by Lilliefors [1967]. For example, for a given sample of N observations, the empirical cumulative distribution function calculated by the California method is given by

$$F(X_i) = \frac{i}{N} \quad (5.18)$$

in which X_i is the i -th largest observed value in the random sample of size N . On the other hand, the hypothesized cumulative normal distribution function, $S(X_i)$, can be obtained with the sample mean and sample standard deviation. If the largest of the absolute values of N differences between $F(X_i)$ and $S(X_i)$ exceeds the critical value, which is obtained from a Monte Carlo calculation shown in Table 1 in Lilliefors's paper, then the hypothesis will be rejected that the observations are from a population with normal distribution. A five percent level of significance of rejecting the hypothesis is used in this study.

Field Methods

The field sorptivity measurement using Talsma's method was conducted along with the infiltration and redistribution measurement described in Chapters Two and Three. The experimental layout for HSPA Sites A, B and C is shown in Figure 5.1. The central grid is for double ring infiltration measurements, and the surrounding grids are the sorptivity measurement plots. At the six other soil locations

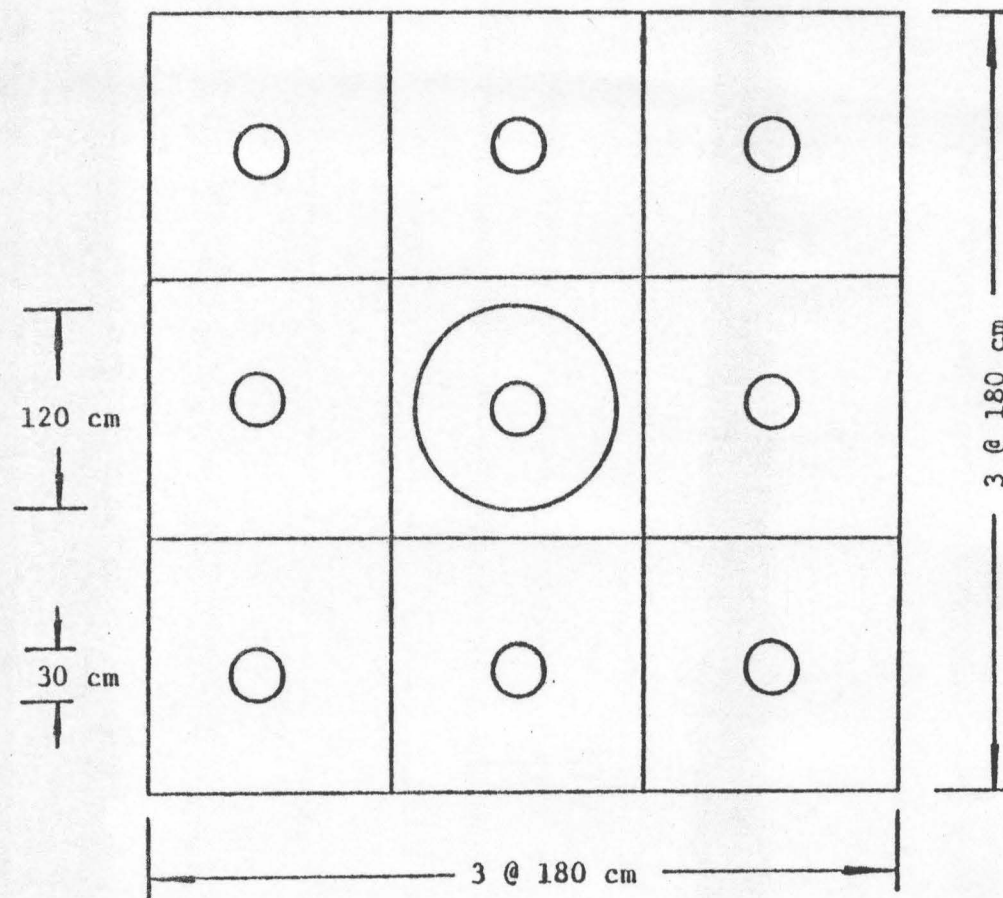


Figure 5.1 Experimental Layout for Infiltration and Sorptivity Measurements.

duplicate sorptivity measurements were made near each of two infiltration sites.

The site preparation for sorptivity measurement was the same as that for infiltration measurement; it involved leveling the soil surface followed by shallow hoeing and final leveling. The infiltrometer ring (30 cm in diameter) was inserted about 15 cm into the soil. The soil was pre-wetted 4 to 5 days before sorptivity was measured.

Just prior to sorptivity measurement, a composite gravimetric antecedent soil sample was obtained from the soil (to about 6 cm deep) within the sorptivity ring with a cork borer (1.5 cm in diameter). After sampling, soil from outside the ring was placed in the resulting hole and compacted.

The method of measuring sorptivity is generally the same as described by Talsma [1969]. Before ponding water into the ring, some porous, fibrous packing material is placed in the ring (only to cover a small area). A known volume of water (1.6 liters) is poured directly on the porous material rather than on the soil to avoid disturbing the soil surface. The subsequent drop in water level was read from a graduated capillary tubing which was inclined at a 9.5 degree angle to the water surface, giving a 6-fold amplification in depth changes with time. The time corresponding to each water-level reading was recorded using a hand-carried digital electronic stopwatch. Normally, the measurement required two people, but one person can handle the entire operation with the help of a tape recorder.

The soil sample for bulk density and particle density was taken within the sorptivity plot after completion of the measurement. The

initial volumetric soil water content for the sorptivity measurement was calculated from the measured gravimetric water content and bulk density. The particle density was determined by the pynometer method described by Blake [1965].

In summary, the experiment was conducted on two different soil series located in seven different areas. At each location infiltration measurements were made at two or three experimental sites, 10 to 20 meters apart. The infiltration measurement was made on dry soil first, followed by a subsequent infiltration run on moist soil about four days later. The antecedent water content of the dry soil ranged from 17 to 27% by volume while the moist soil ranged from 33 to 38%. Totally, 26 infiltration measurements were made in these studies [Green et al., 1979, in preparation].

The sorptivity measurement was made on 3 to 24 experimental sites at each soil location next to the infiltration experimental sites. The results of infiltration and sorptivity measurements will be discussed separately.

RESULTS AND DISCUSSION

Variability of Sorptivity in a Large Area

Brutsaert [1976] stated that the variability of sorptivity in a large area is probably log-normally distributed. This conclusion is drawn based on the equation used to calculate sorptivity. In his equation, sorptivity is a function of K_s . Since K_s is log-normally distributed in a large area [Biggar and Nielsen, 1976; Peck et al., 1977], sorptivity should have the same type of distribution as K_s .

In the results, two types of normality-test results are shown in Table 5.1. One is the normality test on the original data, the other is the test on the data with log-transformation.

Table 5.1 shows that D values which were calculated from the log-transformed data are smaller than those obtained from the original data. Also, the D for log S is less than the critical value at the 5% level, indicating that log S is normally distributed while S is non-normal. These results provide further evidence that field measured sorptivity in a large area is log-normally distributed. Similar results were obtained by an analysis of sorptivity data obtained from other sources (unpublished data). Therefore, a representative field-measured sorptivity in a large area is best estimated by the geometric mean of the local values. Hence, the geometric mean of field-measured sorptivity is used in this study for one point in the linear approximation of $S(\theta_o)$; the other point is $S = 0$ when θ_o is at field saturation.

Calculation of Sorptivity and Prediction of Infiltration

A method of obtaining S versus θ_o by a linear approximation has been described. An example of S versus θ_o is shown in Figure 5.2. The dashed line is the matched $S(\theta_o)$ calculated from $K(\theta)$ and $D(\theta)$ and has been shown in Chapter Three. The solid line represents the linear approximation.

In the calculation of infiltration, K_s in (5.16) is approximated by the geometric mean value of field-measured "steady" flux, i_s (which is essentially the field-measured "steady" infiltration rate). To have $K_s = i_s$ requires the assumption of unit gradient along the profile during the infiltration measurement. The reasons for using the "steady"

Table 5.1 The Results of Kolmogorov-Smirnov Test on the Field-Measured Sorptivity Distribution

Soil Series	Sample Size N	Critical Value at 5% level*	D = Max F(X) - S(X)	
			S	log S
Molokai	33	0.1542	0.1866	0.1411
Lahaina	26	0.1738	0.1867	0.1211
Overall	59	0.1153	0.2049	0.1120

* Calculated by $0.866/\sqrt{N}$, from Lilliefors [1967].

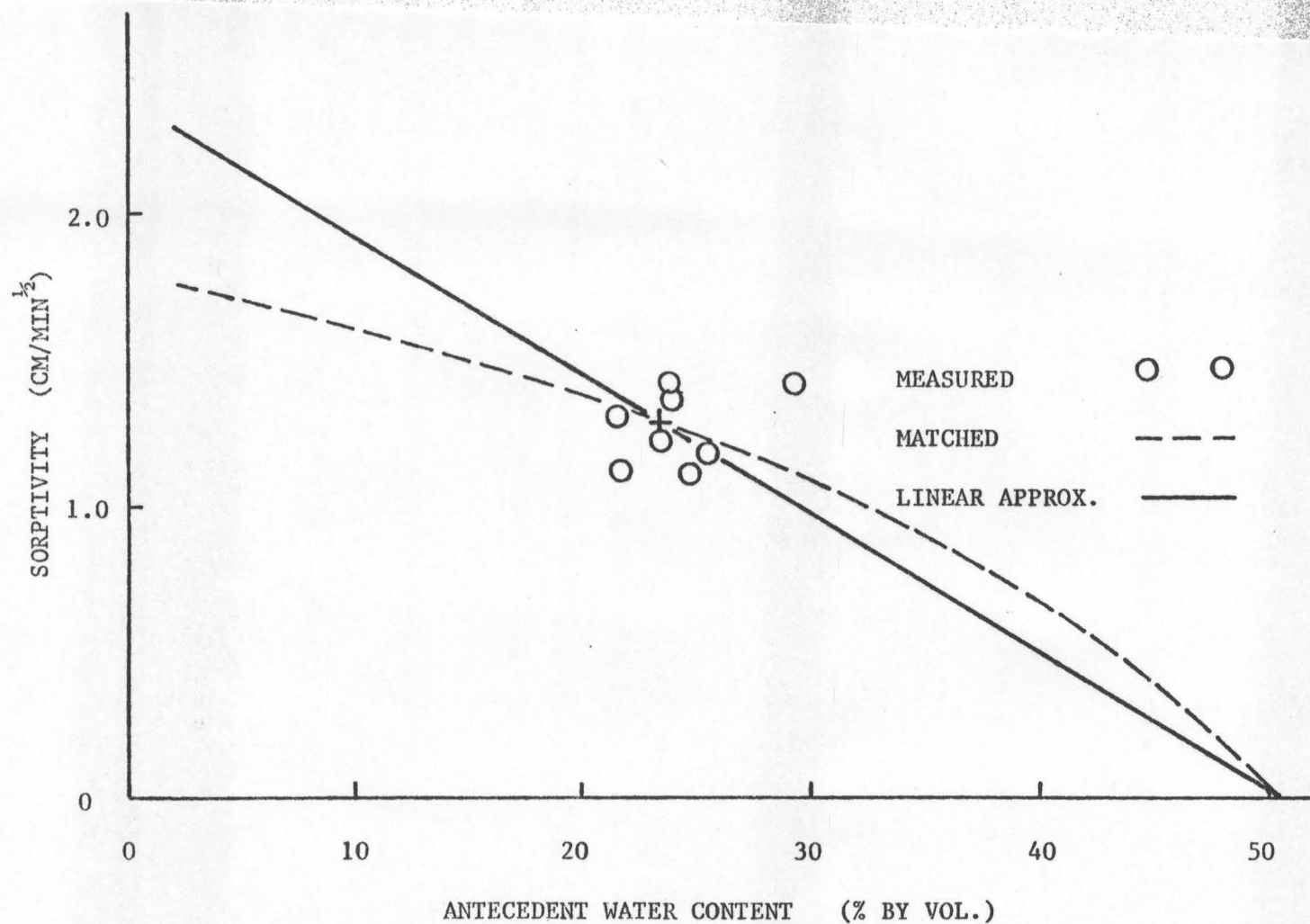


Figure 5.2 Sorptivity As a Function of Antecedent Water Content--A Linear Approximation for HSPA, Site A.

flux to approximate K_s are: firstly, because a tensiometer is required in the experimental site to measure the water potential during infiltration if the true value of K_s is to be obtained. It is more convenient and economical, especially in characterizing an entire watershed, if tensiometers can be eliminated in the method. Secondly, the maximum difference between K_s and i_s for our field measurements was about two-fold (see Table 2.3 in Chapter Two). The error thus contributed to the calculated cumulative infiltration due to using i_s in place of K_s in the third term of the right-hand-side of (5.16) is essentially zero (squaring of K_s reduces the error). In the second term of the right-hand-side of (5.16), the error in this term associated with the estimate of K_s depends upon the magnitude of t . Using HSPA Site A as an example, i_s and S values used to calculate I are 0.0188 cm/min and $1.05 \text{ cm/min}^{1/2}$, respectively. If I is calculated using $K_s = i_s$, $I = 8.53$ cm of water when $t = 60$ min, and if $K_s = 2i_s$, $I = 8.96$ cm. Therefore, the difference between the two computed results is about 5%, which constitutes a reasonably small error for a field method.

Figure 5.3 shows results of calculated and measured cumulative infiltration for elapsed times of 5 minutes to the time, t , when the wetting front is estimated to have reached the B horizon of the profile. The reason for using infiltration from 5 minutes to t for comparisons is given elsewhere (see Chapters Three and Four). Cumulative infiltration results predicted by Equation (5.16) are generally good, although some individual predictions exhibit considerable error. The predicted and measured values are well correlated ($r = 0.93$); the average percentage error for all predicted values is 23%, somewhat larger than the error

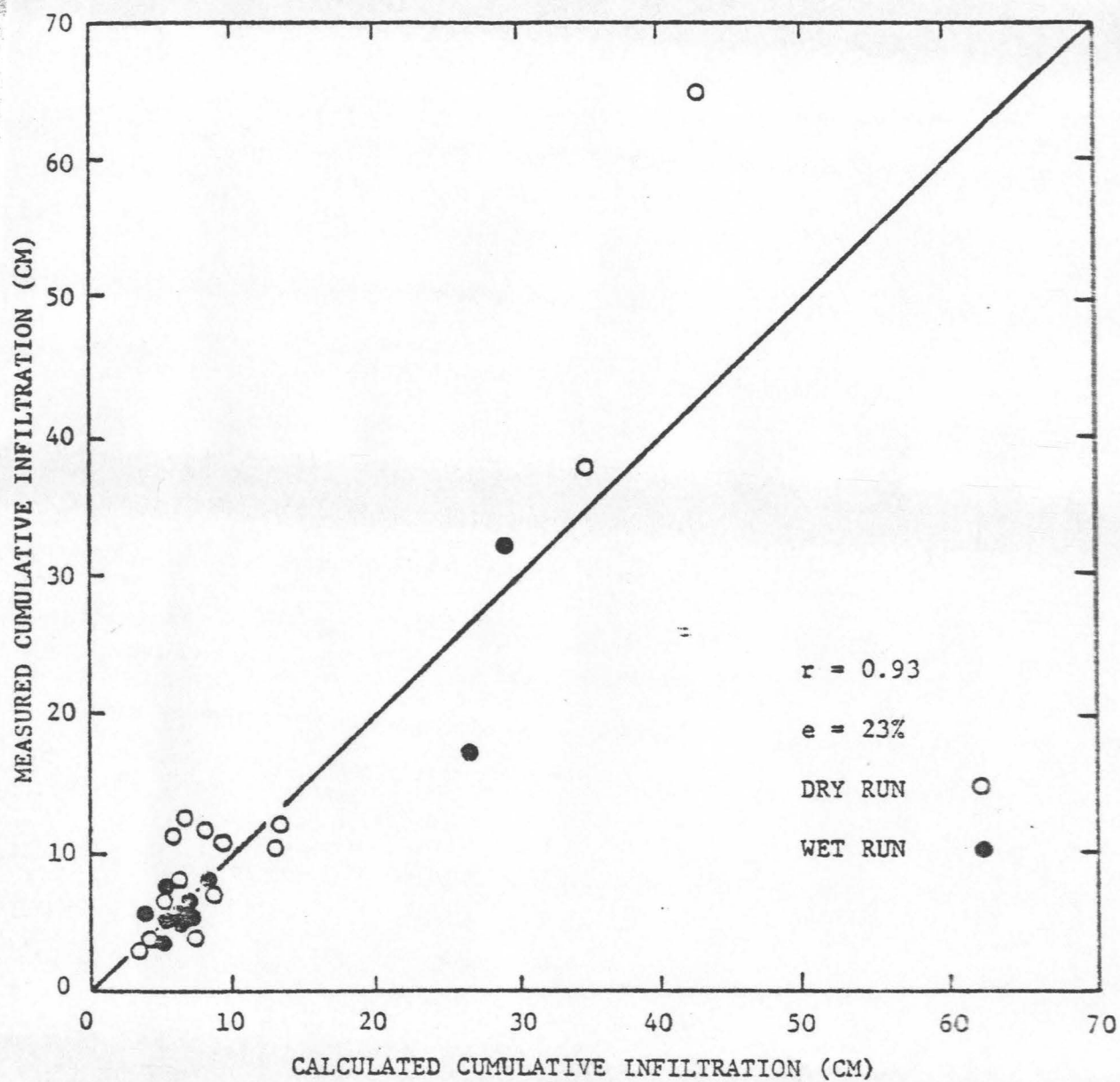


Figure 5.3 Comparison of Calculated and Measured Cumulative Infiltration.

of cumulative infiltration predictions by the Philip 2-term equation based on a more detailed analysis utilizing field drainage data (see Chapter Three). While the prediction method examined in the present paper apparently sacrifices some accuracy, it is extremely simple, requiring only two parameters which are easily measured in the field. The method should allow infiltration prediction at a large number of sites in a watershed and thus provide a means of characterizing spatial variability of infiltration.

SUMMARY AND CONCLUSION

In this study the sorptivity of surface soil was measured using Talsma's method. The statistical distribution of field-measured sorptivity in a large area was found to be log-normal by the Kolmogorov-Smirnov test. A linear relation between S and θ was assumed for the purpose of infiltration prediction. The linear $S(\theta_0)$ relation was obtained from the geometric mean of the field-measured sorptivity and the sorptivity at saturation (assumed to be zero). The infiltration equation of Talsma and Parlange, which requires only the $S(\theta_0)$ relation and saturated hydraulic conductivity, was employed to predict infiltration.

The method was tested on two soil series at seven soil locations, for a total of 26 infiltration measurements, including both "dry" and "wet" antecedent conditions. Predictions of cumulative infiltration by this method were reasonably good, considering the simplicity of the method. More accurate predictions are obtained by a more detailed analysis requiring profile soil-water redistribution data, but the

detailed method is considered too time-consuming and complex for extensive field use.

CHAPTER SIX

OVERALL CONCLUSION

The following conclusions can be drawn from this study:

1. Water content, θ , and water potential, ψ , in the soil profile during the post-infiltration redistribution process have a log-linear relationship with time, allowing field redistribution data to be represented by simple mathematical expressions.
2. The following equations were derived for calculating hydraulic conductivity, K , diffusivity, D , and the wetting front potential, H_f , in this study.

$$K = -L b a \left(\frac{1}{b} \right) \theta \left(\frac{b-1}{b} \right)$$

$$D = -L m n a^{-\left(\frac{n-1}{b} \right)} \theta \left(\frac{n-1}{b} \right)$$

$$H_f = m \left(\frac{\theta_s}{a} \right)^{\frac{n}{b}} \left(\frac{b-1}{b+n-1} \right) \left[\frac{1 - \left(\frac{\theta_i}{\theta_s} \right)^{\frac{b+n-1}{b}}}{1 - \left(\frac{\theta_i}{\theta_s} \right)^{\frac{b-1}{b}}} \right]$$

The proposed methods for calculating K , D and H_f are simple and rapid, and the value of each soil physical property can be obtained for a wide range of soil water contents. Moreover, all of these soil physical properties are measured from a transient-state flow system in the field; thus they should be appropriate for prediction of soil-water movement in the field.

3. The calculated 85% of total porosity obtained from the measured bulk density and particle density is a good approximation of "field-saturated" water content for the soil in this study.
4. The matching factors for $K(\theta)$ and $D(\theta)$ are identical; this identity is shown mathematically.
5. The surface soil (to about 10 cm deep) has a tremendous influence on infiltration, especially on the early portion of infiltration. Talsma's field sorptivity method was used to characterize the surface soil. In the prediction of infiltration, the field-measured sorptivity was found to be a very useful infiltration-related physical soil parameter.
6. Sorptivity and the coefficient A, used in Philip's equation to predict infiltration, were calculated from the $K(\theta)$ and $D(\theta)$ obtained from the method developed in this study (item 2 above). Generally, the predicted infiltration rates are under-estimates, but predictions were greatly improved when the calculated sorptivity is matched to the field-measured value.
7. The wetting front potential is virtually constant over a wide range of antecedent water contents (e.g. when the water content is lower than that at field capacity); this can be explained mathematically by the derived wetting front potential equation.
8. Both the Philip and Green-Ampt approaches of this study can be used to predict infiltration, but with certain constraints. The Green-Ampt approach is good for predicting infiltration on dry soil only. Philip's two-parameter equation is superior to the Green-Ampt equation in watershed infiltration analysis because

Philip's equation predicted infiltration equally well on both wet and dry soil.

9. The statistical distribution of field-measured sorptivity in a large area was found to be log-normal by the Kolmogorov-Smirnov test.
10. The Talsma-Parlange equation, which requires only the sorptivity and saturated hydraulic conductivity, was employed to predict infiltration. A linear $S(\theta_0)$ relation was assumed from the geometric mean of field-measured sorptivity and the sorptivity at saturation (assumed to be zero). Predictions of infiltration by the Talsma-Parlange equation, with the assumption of linear $S(\theta_0)$ relation, are reasonably good, considering the simplicity of the method. More accurate predictions are obtained by the more detailed analyses using field-measured $K(\theta)$ and $D(\theta)$, but the detailed methods are considered too complex and time consuming for routine field use, especially in watershed simulation.
11. In watershed modeling, especially for ungaged watersheds, process models are preferred. Field measurement of appropriate hydrological parameters of soils will afford the greatest single advance in the simulation of the rainfall-runoff process [Chapman, 1975]. Since K_s and S can be measured easily in the field, the linear approximation of S versus θ developed in Chapter Five and application of the Talsma-Parlange equation to predict infiltration should be appropriate for watershed infiltration prediction.
12. The log-linear relationship between θ and t assumed in Chapter Two should be adequate to describe the soil-water redistribution

process or to determine the antecedent soil-water content of the next infiltration occurrence. Therefore, the log-linear relation of θ and t combined with the Talsma-Parlange equation provides a practical method to describe the entire soil-water infiltration-redistribution cycle in a watershed.

APPENDIX I

THE DETAILED DERIVATION OF EQUATIONS

(2.1), (2.2), (2.7) AND (2.8)

From Richard's equation,

$$\frac{\partial \theta}{\partial t} = - \frac{\partial}{\partial z} \left(K \frac{\partial \psi}{\partial z} \right) \quad (\text{A1.1})$$

Integrating (A1.1) with respect to depth, we have

$$\int_0^L \frac{\partial \theta}{\partial t} dz = - \int_0^L \frac{\partial}{\partial z} \left(K \frac{\partial \psi}{\partial z} \right) dz \quad (\text{A1.2})$$

The left hand side (LHS) of (A1.2) can be expressed by Leibnitz's rule as:

$$\begin{aligned} \text{LHS} &= \int_0^L \frac{\partial \theta}{\partial t} dz = \frac{\partial}{\partial t} \int_0^L \theta dz - \theta \Big|_L \frac{\partial L}{\partial t} + \theta \Big|_0 \frac{\partial 0}{\partial t} \\ &= \frac{\partial}{\partial t} \int_0^L \theta dz \\ &= L \frac{\partial}{\partial t} \left[\frac{\int_0^L \theta dz}{L} \right] \\ &= L \frac{\partial \bar{\theta}}{\partial t} \end{aligned} \quad (\text{A1.3})$$

where,

$\bar{\theta}$ is the average soil-water content in the profile from 0 to L cm.

The right hand side of (A1.2) can be written as,

$$- \int_0^L \frac{\partial}{\partial z} \left[K \frac{\partial \psi}{\partial z} \right] dz = - \left[K \frac{\partial \psi}{\partial z} \Big|_L - K \frac{\partial \psi}{\partial z} \Big|_0 \right] \quad (\text{A1.4})$$

During the redistribution period, it is assumed that there is no water flow through the ground surface, i.e.

$$K \frac{\partial \psi}{\partial z} \Big|_0 = 0.$$

Furthermore, for unsaturated flow, the total water potential is equal to the sum of matrix potential and the gravitational potential, i.e.

$$\psi = \psi_M + \psi_g \quad (\text{A1.5})$$

where

ψ_M is matrix potential, cm.
 ψ_g is gravitational potential, cm.

Since the soil profile is assumed to be uniform, and at the same water content throughout, ψ_M should be a constant along the soil profile.

(A1.4) with the substitution of (A1.5) becomes,

$$\begin{aligned} \text{RHS} &= - \left[K \frac{\partial \psi}{\partial z} \Big|_L \right] \\ &= - K \frac{\partial \psi_M}{\partial z} \Big|_L - K \frac{\partial \psi_g}{\partial z} \Big|_L \\ &= - K \frac{\partial \psi_g}{\partial z} \Big|_L \end{aligned}$$

If the datum of z is at the ground surface (i.e. $z = 0$), $\psi_g = z$.

Therefore,

$$\text{RHS} = - K \frac{\partial z}{\partial z} = -K_L \quad (\text{A1.6})$$

Because (A1.3) is equal to (A1.6) (i.e. $\text{RHS} = \text{LHS}$), so that

$$K_L = - L \frac{\partial \bar{\theta}}{\partial t} \quad (\text{A1.7})$$

If an average soil-water characteristic curve holds for the entire soil profile, by the Chain-rule,

$$\frac{\partial \bar{\theta}}{\partial t} = \frac{d\bar{\theta}}{d\psi} \left(\frac{\partial \psi}{\partial t} \right) \quad (\text{A1.8})$$

Now, by the definition of diffusivity,

$$D_L = K_L \frac{d\psi}{d\bar{\theta}} \quad (\text{A1.9})$$

Combining (A1.7), (A1.8) and (A1.9), D can be written as

$$D_L = -L \left(\frac{\partial \bar{\theta}}{\partial t} \right) \frac{\left(\frac{\partial \psi}{\partial t} \right)}{\left(\frac{\partial \bar{\theta}}{\partial t} \right)}$$

and finally,

$$D_L = -L \frac{\partial \psi}{\partial t} \quad (\text{A1.10})$$

If we assume that both $\bar{\theta}$ and ψ are functions of time only, (A1.7) and (A1.10) can be written as

$$K_L = -L \frac{d\bar{\theta}}{dt} \quad (\text{A1.11})$$

$$D_L = -L \frac{d\psi}{dt} \quad (\text{A1.12})$$

(A1.11) and (A1.12) are Equations (2.1) and (2.2) in Chapter Two, respectively.

According to Richards et al. [1956] and Gardner et al. [1970], soil-water content during the redistribution period can be expressed as a function of time, such that

$$\theta = at^b \quad (\text{A1.13})$$

Moreover, it is assumed in this study that ψ also can be expressed as a function of time as

$$\psi = mt^n \quad (\text{A1.14})$$

Substuting (A1.13) and (A1.14) into (A1.11) and (A1.12), we can obtain the following equations:

$$K_L = -L abt^{b-1} \quad (\text{A1.15})$$

$$D_L = -L mnt^{n-1} \quad (\text{A1.16})$$

Again, substituting (A1.13) into (A1.15) and (A1.16) for t , we finally have

$$K_L = -L b a^{\left(\frac{1}{b}\right)} (\theta)^{\left(\frac{b-1}{b}\right)} \quad (\text{A1.17})$$

$$D_L = -L m n a^{-\left(\frac{n-1}{b}\right)} (\theta)^{\left(\frac{n-1}{b}\right)} \quad (\text{A1.18})$$

Equations (A1.17) and (A1.18) are Equations (2.7) and (2.8) in Chapter Two.

APPENDIX II

PROOF OF IDENTITY OF MATCHING FACTORS FOR K AND D

By definition of diffusivity, we have

$$D = K \frac{d\psi}{d\theta} \quad (A2.1)$$

By the Chain-rule

$$D = K \frac{d\psi}{dt} \frac{dt}{d\theta}$$

or

$$D = \frac{K}{\left(\frac{d\theta}{dt}\right)} \frac{d\psi}{dt}$$

$$D = \frac{K}{\left(-L \frac{d\theta}{dt}\right)} \cdot \left(-L \frac{d\psi}{dt}\right)$$

Recall from Appendix I that

$$K_L = -L \frac{d\theta}{dt}$$

and

$$D_L = -L \frac{d\psi}{dt} .$$

Thus

$$D = \frac{K}{K_L} \cdot D_L \quad (A2.2)$$

In Equation (A2.2), K is the true (measured) conductivity and K_L and D_L are, respectively, the conductivity and diffusivity calculated by the simplified method. The matching factor for conductivity is K/K_L as described in Chapter Two and Equation (A2.2) shows that K/K_L is also the appropriate matching factor for the derived diffusivity.

APPENDIX III

DERIVATION OF THE WETTING FRONT POTENTIAL EQUATION

From Mein and Larson [1971]:

$$H_f = \frac{\int_{K_i}^{K_s} \psi dK}{K_s - K_i} \quad (A3.1)$$

From Chapter Two, we have

$$\theta = at^b,$$

$$\psi = mt^n,$$

and

$$K = -L ab \left(\frac{1}{m}\right) \left(\frac{b-1}{n}\right) \psi \left(\frac{b-1}{n}\right) \quad (A3.2)$$

Let

$$C_1 = -L ab \left(\frac{1}{m}\right) \left(\frac{b-1}{n}\right)$$

$$C_2 = \left(\frac{b-1}{n}\right)$$

Therefore, (A3.2) becomes

$$K = C_1 \psi^{C_2} \quad (A3.3)$$

Substituting (A3.3) into (A3.1), we have

$$H_f = \frac{1}{K_s - K_i} \int_{K_i}^{K_s} \left(\frac{K}{C_1}\right)^{\frac{1}{C_2}} dK$$

$$\begin{aligned}
&= \left(\frac{1}{C_1}\right)^{\frac{1}{C_2}} \left(\frac{C_2}{C_2+1}\right) \left[\frac{K_s \left(\frac{1}{C_2}+1\right) - K_i \left(\frac{1}{C_2}+1\right)}{K_s - K_i} \right] \\
&= \left(\frac{1}{C_1}\right)^{\frac{1}{C_2}} \left(\frac{C_2}{C_2+1}\right) K_s^{\frac{1}{C_2}} \frac{\left[1 - \left(\frac{K_i}{K_s}\right)^{\left(\frac{1}{C_2}+1\right)} \right]}{\left[1 - \frac{K_i}{K_s} \right]} \quad (A3.4)
\end{aligned}$$

Let K_* by the matched K which is a function of water content
(the K that we used in calculation),

$$K_* = K_c \cdot \frac{K_{sm}}{K_{sc}} \quad (A3.5)$$

where

K_c is the calculated K which is a function of water content
(from Equation (2.7)).

K_{sm} is the measured "field saturated" K .

K_{sc} is the calculated "field saturated" K .

Therefore, the relative hydraulic conductivity, K_r , is equal to

$$\begin{aligned}
K_r &= \frac{K_*}{K_* \text{ at saturated}} = \frac{K_*}{K_{sm}} \\
&= \frac{K_c \cdot \frac{K_{sm}}{K_{sc}}}{K_{sm}} \\
K_r &= \frac{K_c}{K_{sc}} = \frac{\text{Calculated } K}{\text{Calculated saturated } K} \quad (A3.6)
\end{aligned}$$

Note that the measured K_{sm} is implicitly accounted for in the calculation but is no longer in (A3.6).

$$K_r = \frac{K_c}{K_{sc}} = \frac{-L b a \left(\frac{1}{b}\right) (\theta_i) \left(\frac{b-1}{b}\right)}{-L b a \left(\frac{1}{b}\right) (\theta_s) \left(\frac{b-1}{b}\right)}$$

$$K_r = \left(\frac{\theta_i}{\theta_s}\right)^{\left(\frac{b-1}{b}\right)} \quad (A3.7)$$

Substituting (A3.7) into (A3.4) with $K_r = K_i/K_s$, we have

$$H_f = \left(\frac{K_{sc}}{C_1}\right)^{\frac{1}{C_2}} \left(\frac{C_2}{C_2+1}\right) \frac{\left[1 - \left(\frac{\theta_i}{\theta_s}\right)^{\left(\frac{b+n-1}{b}\right)}\right]}{\left[1 - \left(\frac{\theta_i}{\theta_s}\right)^{\left(\frac{b-1}{b}\right)}\right]} \quad (A3.8)$$

Substituting (A3.2) into (A3.8), finally we have

$$H_f = m \left(\frac{\theta_s}{a}\right)^{\frac{n}{b}} \left(\frac{b-1}{b+n-1}\right) \frac{\left[1 - \left(\frac{\theta_i}{\theta_s}\right)^{\left(\frac{b+n-1}{b}\right)}\right]}{\left[1 - \left(\frac{\theta_i}{\theta_s}\right)^{\frac{b-1}{b}}\right]} \quad (A3.9)$$

Equation (A3.9) is Equation (4.9) in Chapter Four.

APPENDIX IV

EXAMPLE OF THE CALCULATION FOR THE DETERMINATION OF THE FIELD MEASURED SORPTIVITY DISTRIBUTION USING KOLMOGOROV-SMIRNOV TEST

Original Data (1)	Sorted Data (2)	Ranked Number (3)	M/N (4)	Zero Mean Value (5)	Normal Prob Distrib (6)	$ D = (4) - (6) $ (7)
1.29	1.01	1	0.03030	-1.27207	0.10167	0.07137
1.34	1.02	2	0.06061	-1.24769	0.10607	0.04547
1.24	1.03	3	0.09091	-1.22331	0.11061	0.01971
1.38	1.12	4	0.12121	-1.00391	0.15771	0.03651
1.46	1.14	5	0.15152	-0.95516	0.16975	0.01825
1.12	1.14	6	0.18182	-0.95516	0.16975	0.01205
1.14	1.15	7	0.21212	-0.93078	0.17598	0.03612
1.43	1.24	8	0.24242	-0.71138	0.23842	0.00398
1.52	1.29	9	0.27273	-0.58949	0.27777	0.00507
1.82	1.31	10	0.30303	-0.54073	0.29435	0.00865
1.85	1.34	11	0.33333	-0.46760	0.32004	0.01326
2.14	1.34	12	0.36364	-0.46760	0.32004	0.04356
2.53	1.37	13	0.39394	-0.39447	0.34662	0.04728
1.58	1.38	14	0.42424	-0.37009	0.35566	0.06854
1.37	1.43	15	0.45455	-0.24820	0.40199	0.05251
1.83	1.43	16	0.48485	-0.24820	0.40199	0.08281
1.44	1.44	17	0.51515	-0.22382	0.41145	0.10365
1.46	1.46	18	0.54545	-0.17507	0.43051	0.11489
1.34	1.46	19	0.57576	-0.17507	0.43051	0.14519
1.03	1.48	20	0.60606	-0.12631	0.44974	0.15626
2.04	1.48	21	0.63636	-0.12631	0.44974	→ 0.18656
1.43	1.52	22	0.66667	-0.02880	0.48851	0.17809
1.48	1.58	23	0.69697	0.11747	0.54676	0.15014
1.48	1.64	24	0.72727	0.26374	0.60401	0.12319
2.55	1.65	25	0.75758	0.28811	0.61337	0.14413
1.65	1.82	26	0.78788	0.70254	0.75883	0.02897
1.15	1.83	27	0.81818	0.72691	0.76636	0.05174
2.34	1.85	28	0.84848	0.77567	0.78103	0.06737
1.31	2.04	29	0.87879	1.23885	0.89230	0.01360
1.64	2.14	30	0.90909	1.48263	0.93091	0.02191
1.02	2.34	31	0.93939	1.97018	0.97559	0.03629
1.01	2.53	32	0.96970	2.43336	0.99252	0.02292
1.14	2.55	33	1.00000	2.48212	0.99347	0.00643

Mean value of the field measured sorptivity $\bar{x} = 1.532$

Standard deviation of the field measured sorptivity $\sigma = 0.410$

Max $|D| = 0.1866$

Column (1): Field measured sorptivity ($\text{cm}/\text{min}^{1/2}$), x .

Column (4): $N = 33$.

Column (5): Zero mean value $Z = \frac{x - \bar{x}}{\sigma}$.

Column (6): Calculated by $P(Z) = 0.5 + \frac{1}{2} \operatorname{erf}\left(\frac{Z}{\sqrt{2}}\right)$.

Column (7): The absolute value of Column (4) - Column (6).

APPENDIX V

EXAMPLE OF SOLVING GREEN-AMPT EQUATION

Green-Ampt Equation:

$$Kt = I - \Delta\theta(H_o - H_f) \ln \left[1 + \frac{I}{\Delta\theta(H_o - H_f)} \right] \quad (\text{AV.1})$$

Using HSPA Site A dry run for example (see Table 4.2).

Given: $K = 0.0411 \text{ cm/min}$
 $\Delta\theta = 0.224$
 $H_o = 2.0 \text{ cm}$
 $H_f = -34.50 \text{ cm}$

Find ΔI : ΔI is the difference of I at 5 minutes and 49 minutes.

Substituting the given values into (AV.1), we have

$$0.0411t = I - 8.176 \ln \left[1 + \frac{I}{8.176} \right]$$

$$\text{LHS} = 0.0411t \quad (\text{AV.2})$$

$$\text{RHS} = I - 8.176 \ln \left[1 + \frac{I}{8.176} \right] \quad (\text{AV.3})$$

If we assume a series of I values in (AV.3) we will have a series of appropriate RHS values, which can be tabulated as shown in Table AV.1.

Table AV.1

I Versus RHS

I	RHS
0	0
1	0.0566
2	0.2109
3	0.4445
4	0.7438
5	1.0985
6	1.5004
7	1.9430
8	2.4213

Plot I versus RHS and LHS as shown in Figure (AV.1). When $t = 5$ minutes, $I = 1.95$. When $t = 49$ minutes, $I = 7.12$; therefore, $\Delta I = 7.12 - 1.95 = 5.17$ cm.

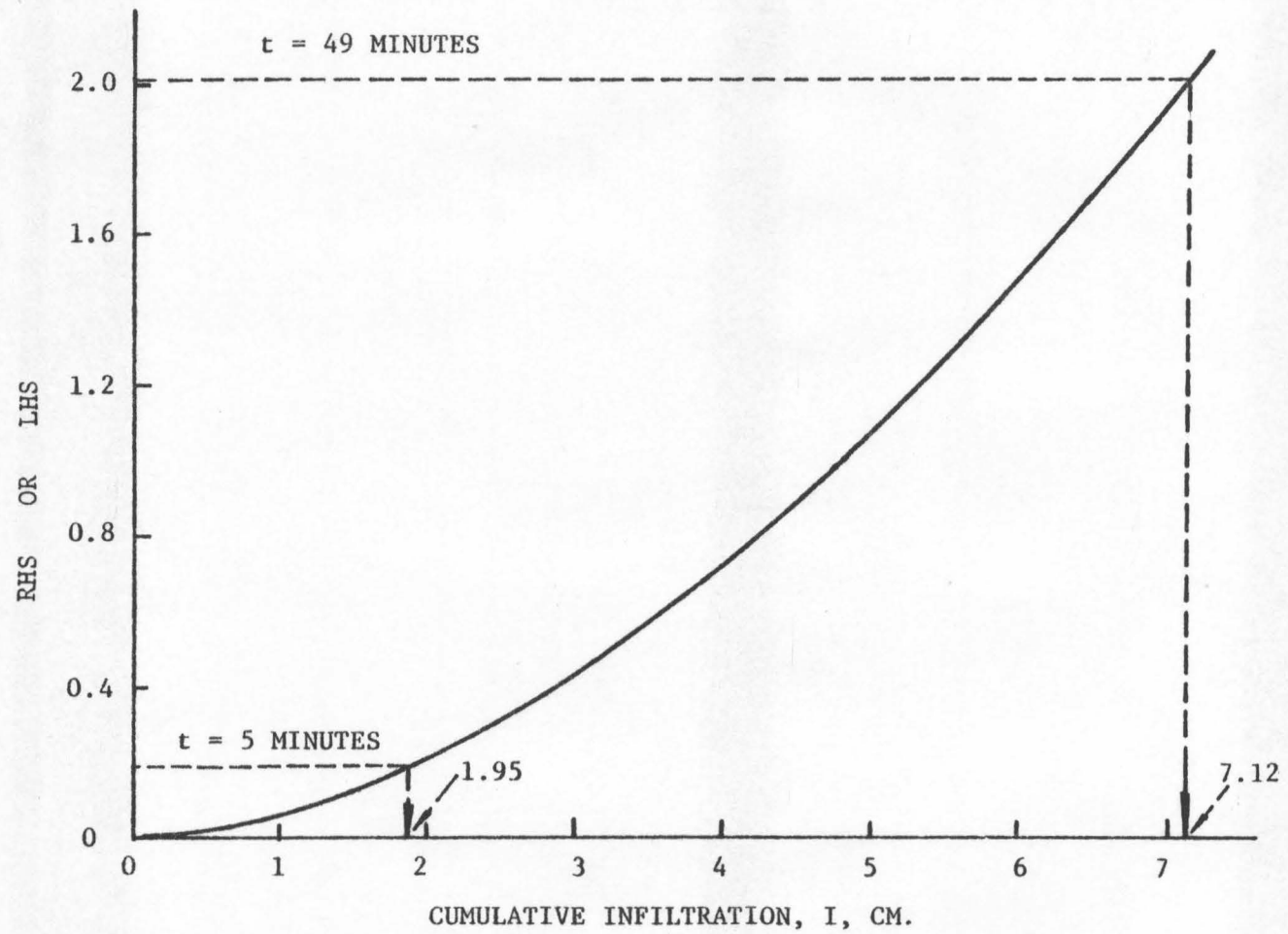


Figure AV.1 Illustration of Solving Cumulative Infiltration, I, in Green-Ampt Equation for HSPA Site A Dry Infiltration Run.

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